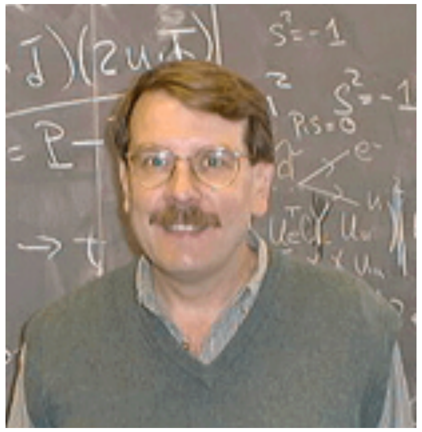
# Great moment in science history 30 years ago




PHYSICAL REVIEW LETTERS

## Amplitude for *n*-Gluon Scattering

OBSERVATION OF JETS IN HIGH TRANSVERSE ENERGY EVENTS
AT THE CWERN PROTON ANTIPROTON COLLIDER
UA1 Collaboration, CERN, Geneva, Switzerland

Stephen J. Parke and T. R. Taylor
*Fermi National Accelerator Laboratory, Batavia, Illinois 60510*

(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

# From the discovery of the gluon jet to the NNLO precision for multi-jet production

Zoltan Kunszt
ITP, ETH, Zurich

MHV@30:
Amplitudes and Modern Applications

March 16, 2016
Fermilab

# Outline

Historical overview ( 1979-1991, 1992-2003, 2004-2015 )

hoping to increase enthusiasm in striving for the automation of calculating multi-jet cross-sections with NNLO accuracy

# Observations of jet production ( 1979-1983 )

1) As a result of important experimental developments (1979-1983) by 1983 it was clear that hard processes with multi jet production are very important observables at high energy colliders

   i) Discovery of gluon jets at PETRA (DESY)

   ii) Observations of jets at the SPS by UA1,UA2, (CERN, 1982)

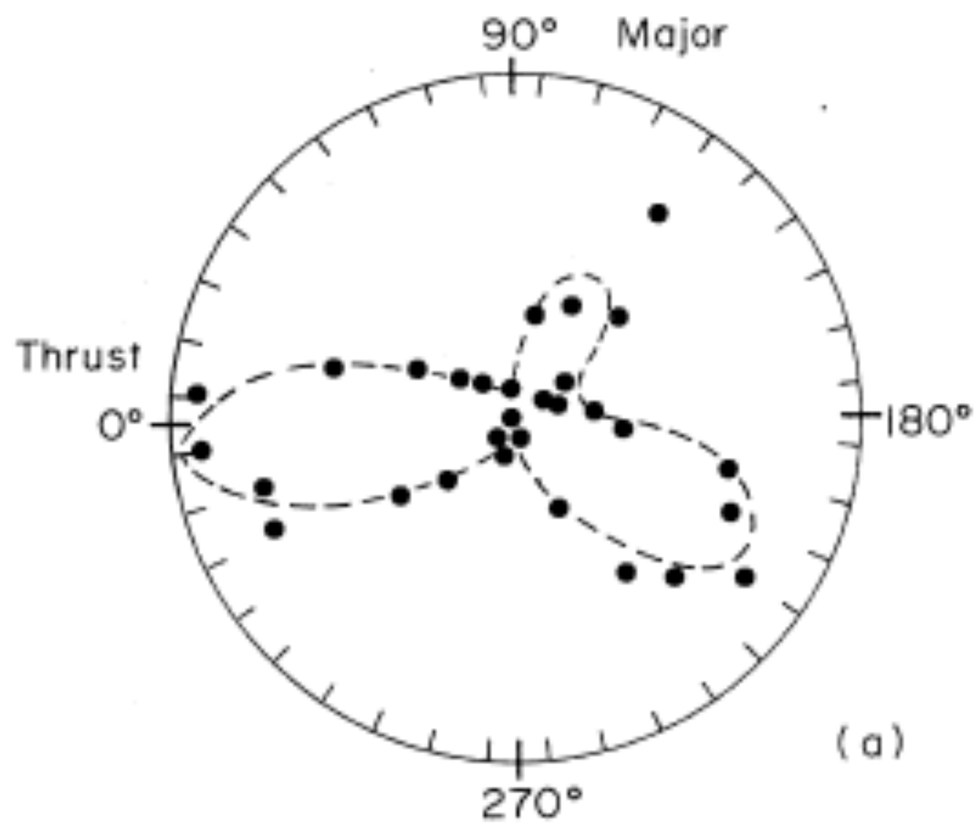
   iii) Proposals for building large hadron colliders SSC and LHC (1983)

2) QCD perturbation theory likely gives reliable predictions, we need to calculate the matrix elements of a very large number of 4, ,6,.. leg processes

3) First calculations of multi-jet processes processes for high energy hadron colliders

# Discovery of gluon jets at PETRA (DESY)

The public announcement of the gluon discovery came at the Lepton/Photon Symposium held at Fermilab in August 1979.



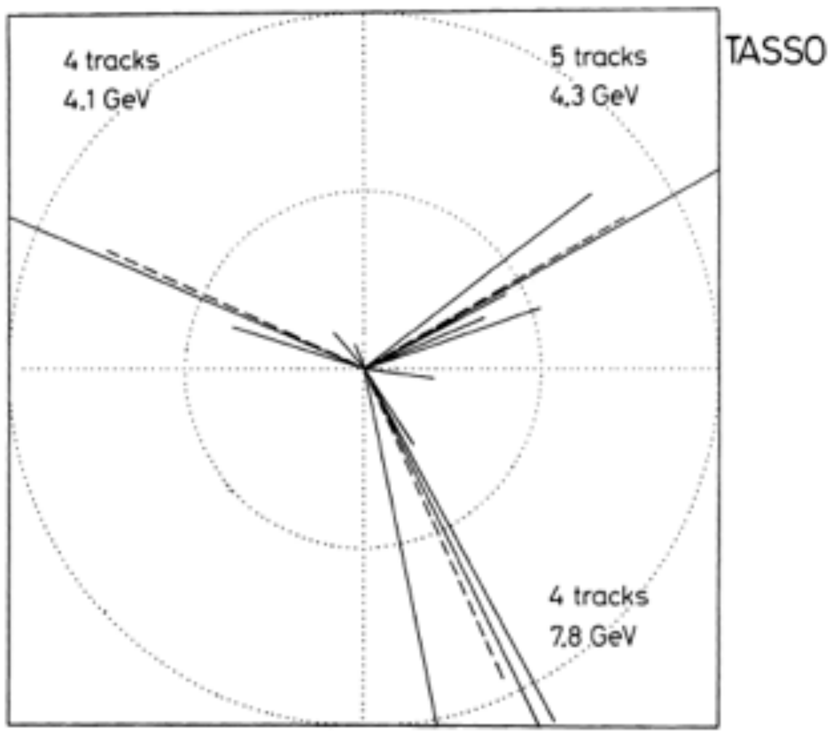




Fig.20g Another 3-jet event projected into the event plane.

Discovery of Three-Jet Events…Mark-J
Phys. Rev. Lett. 43, 830 – 17
September 1979

Evidence for Planar Events…, TASSO
Published in Phys.Lett. B86 (1979)
243

# Observations of jets at the SPS by UA1,UA2

CERN, 1982,1983

Volume 118B, number 1, 2, 3 PHYSICS LETTERS 2 December 1982

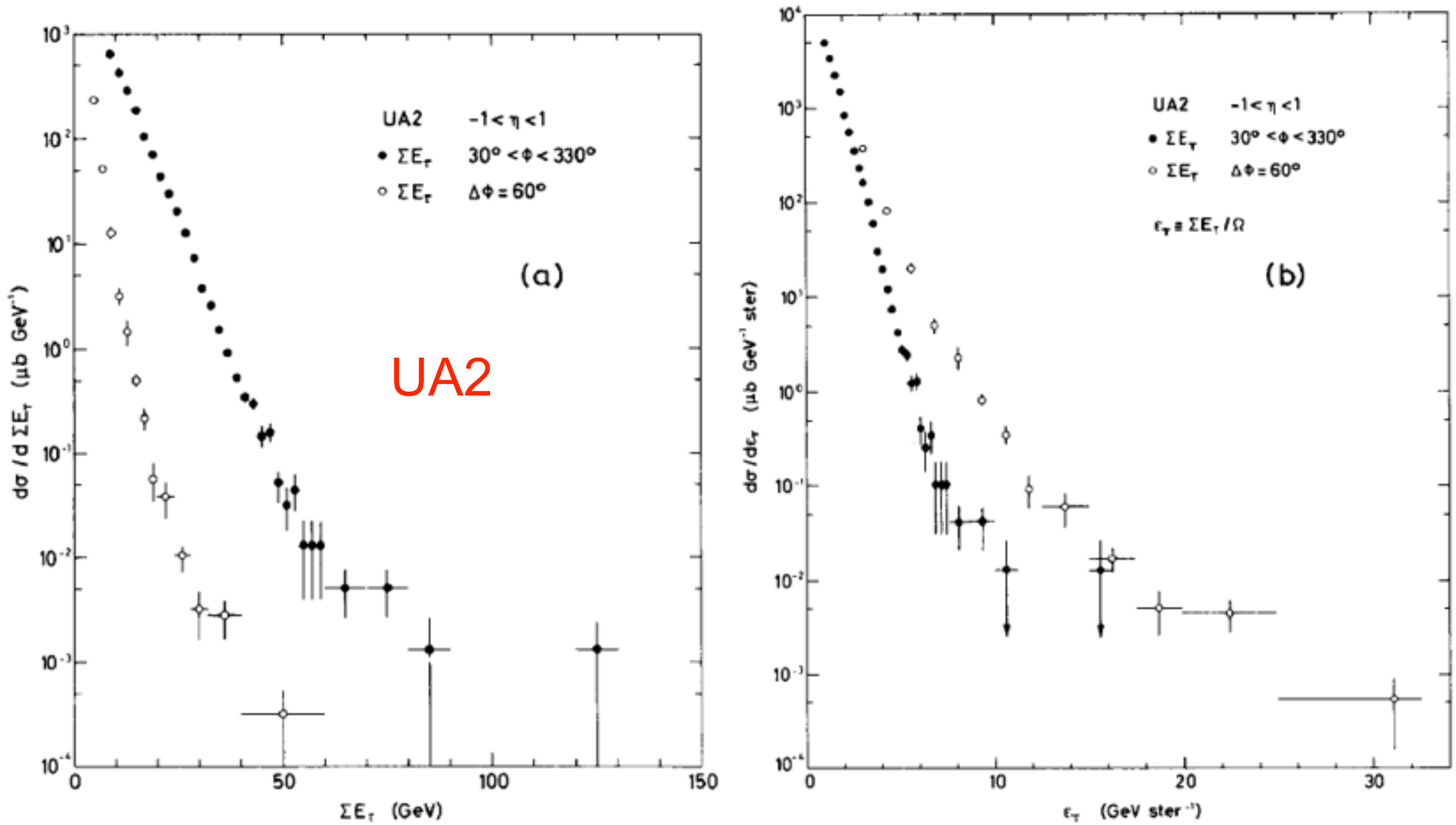



Fig. 2. Transverse energy (a) and its average density (b): observed distributions over the whole azimuthal acceptance (full dots) and over a restricted azimuthal region, $\Delta\phi = 60°$ (open circles).

Similar plots by UA1

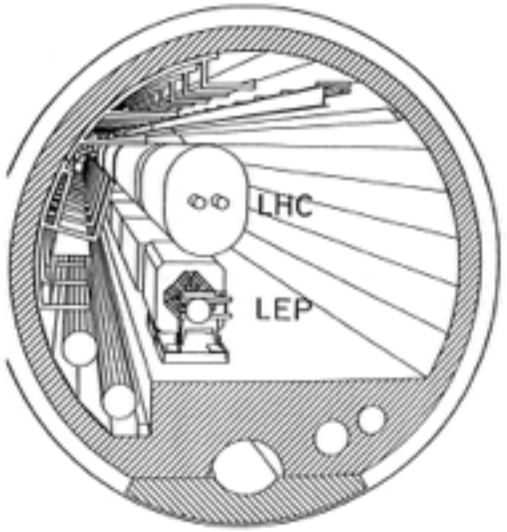
# Proposals for building large hadron colliders SSC and LHC (1983)

Wojcicki, S., J. Adams, T. Appelquist, C. Baltay, M. K. Gaillard, J. D. Jackson, D. Keefe, A. Kerman, L. Pondrom, J. Rees, C. Rubbia, F. Sciulli, M. Tigner, S. Treiman, J. Vander Velde, H. Williams, and B. Winstein, 1983, *Report of the 1983 Subpanel on New Facilities for the U.S. High Energy Physics Program of the High Energy Physics Advisory Panel* (U.S. Department of Energy, Washington, D.C.).

**E. Eichten, I. Hinchliffe, Kenneth D. Lane, C. Quigg (Fermilab). Feb 1984. Rev.Mod.Phys. 56 (1984) 579-707**

PROCEEDINGS OF THE ECFA–CERN WORKSHOP

held at Lausanne and Geneva,
21–27 March 1984



# Jets from QCD

J. Ellis, M. K. Gaillard, G. G. Ross (1976) — Production of gluon jets

QCD predictions for the production of quarks and gluons can be taken seriously at sufficiently high energy if reinterpreted in terms of jets

G. Sterman, S. Weinberg (1977) — Infrared safe jet cross section

We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

$$\sigma(E,\theta,\Omega,\epsilon,\delta)=(d\sigma/d\Omega)_0\,\Omega[1-(g_E{}^2/3\pi^2)(3\ln\delta+4\ln\delta\ln 2\epsilon+\pi^2/3-\tfrac{5}{2})]$$

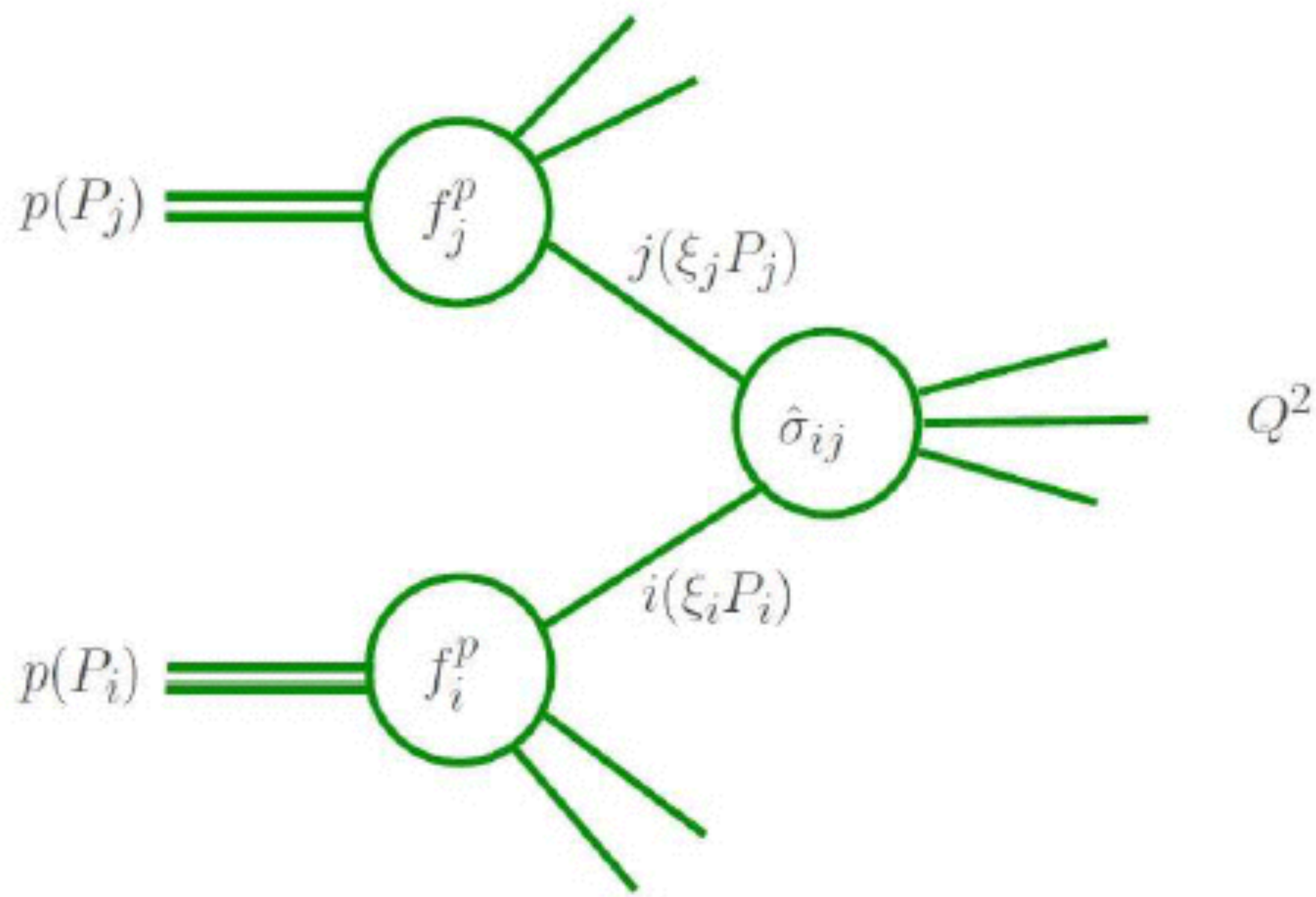
R. P. Feynman and R. D. Field (1977) — A parametrization of the properties of quark jets

G. Altarelli, G. Parisi, (1977) — Asymptotic freedom in parton language

# QCD perturbation theory has been expected to give reliable predictions also for matrix elements of 4,5,6,..leg processes….

R.K. Ellis , H, Georgi, M, Machacek, H.D. Politzer, G. Ross Nucl.Phys. B152 (1979) 285

Collins, Soper, Sterman…




$$d\sigma = d\sigma_{\rm LO} + \left(\frac{\alpha_S}{2\pi}\right) d\sigma_{\rm NLO} + \left(\frac{\alpha_S}{2\pi}\right)^2 d\sigma_{\rm NNLO} + \ldots$$

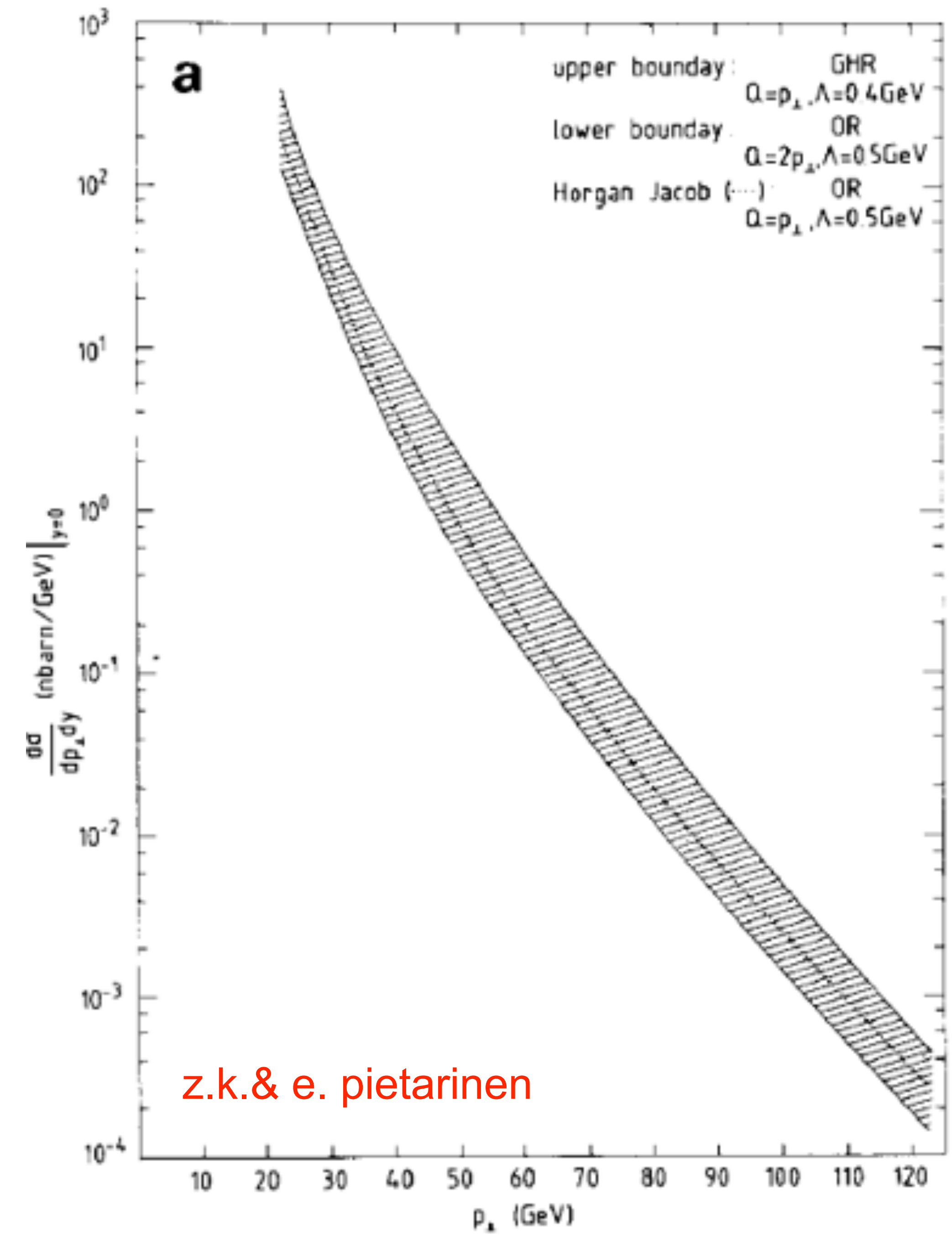
$$d\sigma_{\rm LO}$$

# LO calculations of 2 and 3 jet production

2jets Combridge et.al.(1977), 3jets kz& Pietarinen (1979), 4jets ($e^+e^-$ ) Ali et.al.(1979),

Leading order phenomenology: predictions have very large theoretical error…
scale dependence, parton number densities, jet definitions

| | |
|---|---|
| $q_1 q_2 \to q_1 q_2$, $q_1 \bar{q}_2 \to q_1 \bar{q}_2$ | $\frac{4}{9}\frac{s^2+u^2}{t^2}$ |
| $q_1 q_1 \to q_1 q_1$ | $\frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}\right)-\frac{8}{27}\frac{s^2}{ut}$ |
| $q_1 \bar{q}_1 \to q_2 \bar{q}_2$ | $\frac{4}{9}\frac{t^2+u^2}{s^2}$ |
| $q_1 \bar{q}_1 \to q_1 \bar{q}_1$ | $\frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{t^2+u^2}{s^2}\right)-\frac{8}{27}\frac{u^2}{st}$ |
| $q\bar{q} \to gg$ | $\frac{32}{27}\frac{u^2+t^2}{ut}-\frac{8}{3}\frac{u^2+t^2}{s^2}$ |
| $gg \to q\bar{q}$ | $\frac{1}{6}\frac{u^2+t^2}{ut}-\frac{3}{8}\frac{u^2+t^2}{s^2}$ |
| $qg \to qg$ | $-\frac{4}{9}\frac{u^2+s^2}{us}+\frac{u^2+s^2}{t^2}$ |
| $gg \to gg$ | $\frac{9}{2}\left(3-\frac{ut}{s^2}-\frac{us}{t^2}-\frac{st}{u^2}\right)$ |



# CALKUL collaboration: all five leg processes involving gluons, quarks, leptons and photons with helicity method

F. A. Berends, R. Kleiss , P. De Causmaecker, R. Gastmans , T. T. Wu, (1981)

Calculate amplitudes of fixed helicity helicity configuration and choose special forms of gluon (photon) helicities

$$\hat{\epsilon}^{\pm}(p) = N[\hat{p}\hat{k}\hat{q}(1 \pm \gamma_5) - \hat{k}\hat{q}\hat{p}(1 \pm \gamma_5) \mp (q.k)\hat{p}\gamma_5]$$

## $g(k_1) + g(k_2) \to g(k_3) + g(k_4) + g(k_5)$

$$|\mathrm{M}|^2 = \frac{\mathrm{g}^6}{2} \frac{\mathrm{N}^3}{(\mathrm{N}^2 - 1)} \Big[ \sum_{\mathrm{perms}(2,3,4,5)} (12345) \Big] \frac{\sum_{\mathrm{i<j}} (\mathrm{k_i k_j})^4}{\prod_{\mathrm{i<j}} (\mathrm{k_i k_j})}$$

$$(12345) = (k_1 k_2)(k_2 k_3)(k_3 k_4)(k_4 k_5)(k_5 k_1) \qquad (18)$$

Note the absence of double poles and the factorised form !

Note the simplicity of expression which is the result of the evaluation of 25 Feynman diagram !

# Three improvements for the CALCUL method

- Simpler choice for the gluon helicity vector (Xu,Zhang,Chang(1984/85))

$$\epsilon_\mu^{\pm}(k,p) = \pm \frac{\langle k\pm|\,\gamma_\mu\,|p\pm\rangle}{\sqrt{2}\,\langle p\mp|\,p\pm\rangle}$$

- Manifest crossing symmetry (Gunion,ZK; Kleiss, Stirling (1985))
- Use of N=1 or N=2 supersymmetry (ZK; Parke,Taylor (1985))

Calculations of all 6-parton amplitudes involving gluons,quarks, leptons and gauge bosons (Gunion,ZK; Kleiss,Stilring; Parke,Taylor (1985))

# All 6-leg QCD amplitudes are calculated

Four Jet Processes: Gluon-gluon Scattering To Nonidentical Quark - Anti-quark Pairs
J.F. Gunion , ZK . May 1985.

Gluonic Two Goes to Four , Stephen J. Parke, T.R. Taylor (Fermilab). Aug 1985.

Combined Use of the Calcul Method and N=1 Supersymmetry to Calculate QCD
Six Parton  Processes,  ZK  Nov 1985. (6g,4q2g)

The Cross-Section for Hard Processes Involving Two Quarks and Four
Gluons,  S.J. Parke, T.R. Taylor . Nov 1985.

Six Quark Subprocesses in {QCD}, J. F. Gunion, ZK,  Apr 1986.

Phenomenology: Four Jet Production At Hadron Colliders
Z. K. and  W.J.  Stirling . Jan 1986

Theory: An Amplitude for n Gluon Scattering
S. J. Parke, T.R. Taylor. Mar 1986

# An Amplitude for n Gluon Scattering

In this short Letter, we give a nontrivial squared helicity amplitude for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors. To our knowledge this is the first time in a non-Abelian gauge theory that a nontrivial, on-mass-shell, squared Green's function has been written down for an arbitrary number of external points.

let us make it clear that this result is an educated guess which we have compared to the existing computations and verified by a series of highly nontrivial and nonlinear consistency checks.

S. Parke and T. Taylor, 1986

$$|\mathcal{M}_n(+++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$

$$|\mathcal{M}_n(-++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$

$$|\mathcal{M}_n(--+++\cdots)|^2 = c_n(q,N)[(p_1\cdot p_2)^4 \times \sum_P [(p_1\cdot p_2)(p_2\cdot p_3)(p_3\cdot p_4)\cdots(p_n\cdot p_1)]^{-1} + O(N^{-2}) + O(g^2)],$$

It agrees with known result for n=4,5,6 ( for n=6 numerically)
No double poles, correct behaviour for collinear configurations,…

- How to derive the analytic result for n=6?
- How to derive it for any n?
- Are there other similarly simple amplitudes?
- Phenomenological significance?

# Spinor helicity method and color ordered sub-amplitudes

- How to derive the analytic result for n=6?

The Six Gluon Process as an Example of Weyl-Van Der Waerden Spinor Calculus
F. A. Berends, W. Giele, Jun 30, 1987.

$$\mathcal{M}^{a_1 \cdots a_n}(k_1,\ldots,k_n;\lambda_1,\ldots,\lambda_n)$$
$$=\alpha_n \sum_{P/C(1\cdots n)}^{\frac{1}{2}(n-1)!} \mathrm{Tr}(F^{a_1}\cdots F^{a_n})\mathscr{C}(k_1,\ldots,k_n;\lambda_1,\ldots,\lambda_n),$$

Duality and Multi - Gluon Scattering ,M. L. Mangano, S. J. Parke, Zhan Xu . Jun 1987.

For the *six* gluon scattering process we give explicit and simple expressions for the amplitude and its square. To achieve this we use an analogy with string theories to identify a unique procedure for writing the multi-gluon scattering amplitudes in terms of a sum of gauge invariant dual sub-amplitudes multiplied by an appropriate color (Chan-Paton) factor. The sub-amplitudes defined in this way are invariant under cyclic permutations, satisfy powerful identities which relate different non-cyclic permutations and factorize in the soft gluon limit, the two gluon collinear limit and on multi-gluon poles. Also, to leading order in the number of colors these sub-amplitudes sum *Incoherently* in the square of the full matrix element. The results contained here are important for Monte-Carlo studies of multi-jet processes at hadron colliders as well as for understanding the general structure of QCD.

# Berends-Giele recursion relations

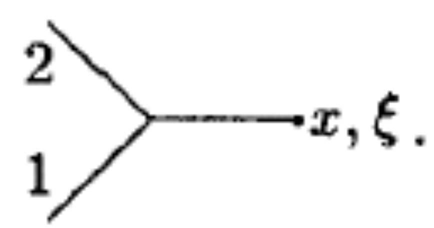
- How to derive Parke-Taylor formula for any n?

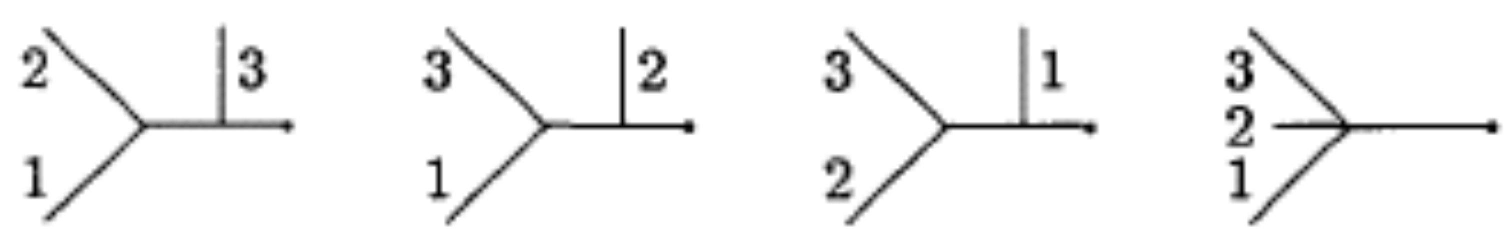
Recursive Calculations for Processes with n Gluons , F. A. Berends, W.T. Giele. Dec 1987.

$$\mathcal{M}(1,\ldots,n) \sim \sum_{P(1,\ldots,n-1)} \mathrm{Tr}(T^{a_1}\ldots T^{a_n})\mathscr{C}(1,\ldots,n),$$

$$\hat{J}_\xi^x(1,2,\ldots,n) = 2g^{n-1} \sum_{P(1,2,\ldots,n)} (a_1 a_2 \ldots a_n x) J_\xi(1,2,\ldots,n).$$

$$J(1,2,\ldots,n) = \frac{1}{\kappa(1,n)^2}\left(\sum_{m=1}^{n-1}[J(1,\ldots,m),J(m+1,\ldots,n)] + \sum_{m=1}^{n-2}\sum_{k=m+1}^{n-1}\{J(1,\ldots,m),J(m+1,\ldots,k),J(k+1,\ldots,n)\}\right)$$



# Berends-Giele recursion relations

Recursive approach gives efficient numerical evaluation.

| $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| # of diagrams | 4 | 25 | 220 | 2485 | 34300 | 559405 | 10525900 |

The requirement of the correct collinear limit, unitarity and N=4 supersymmetry also completely defines the tree MHV amplitudes.

# BG recursion for quark currents

Are there other similarly simple amplitudes?

Conjectures by Mangano and Parke for additional multi gluon amplitudes:

$q\bar{q} + n$ gluons $\quad q\bar{q}q\bar{q} + n$ gluons $\quad V + q\bar{q} + n$ gluons

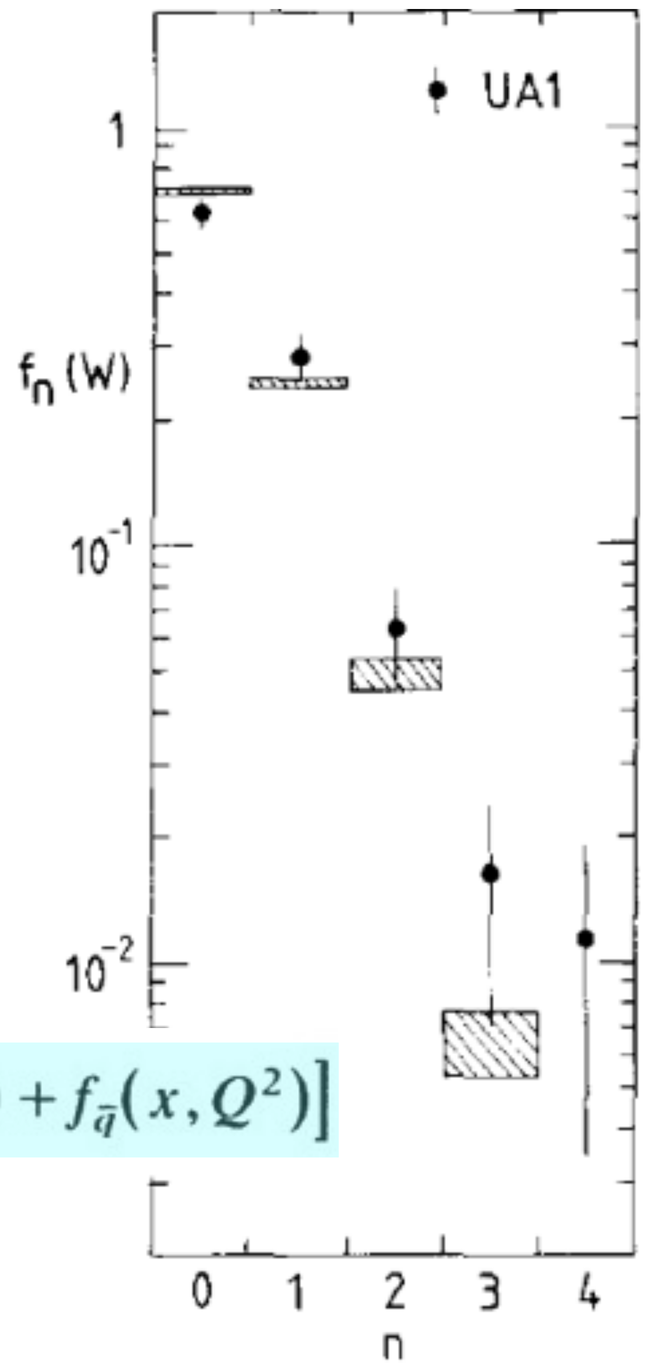
All are proven to be true using BG recursion relations for quark currents by Berends and Giele.

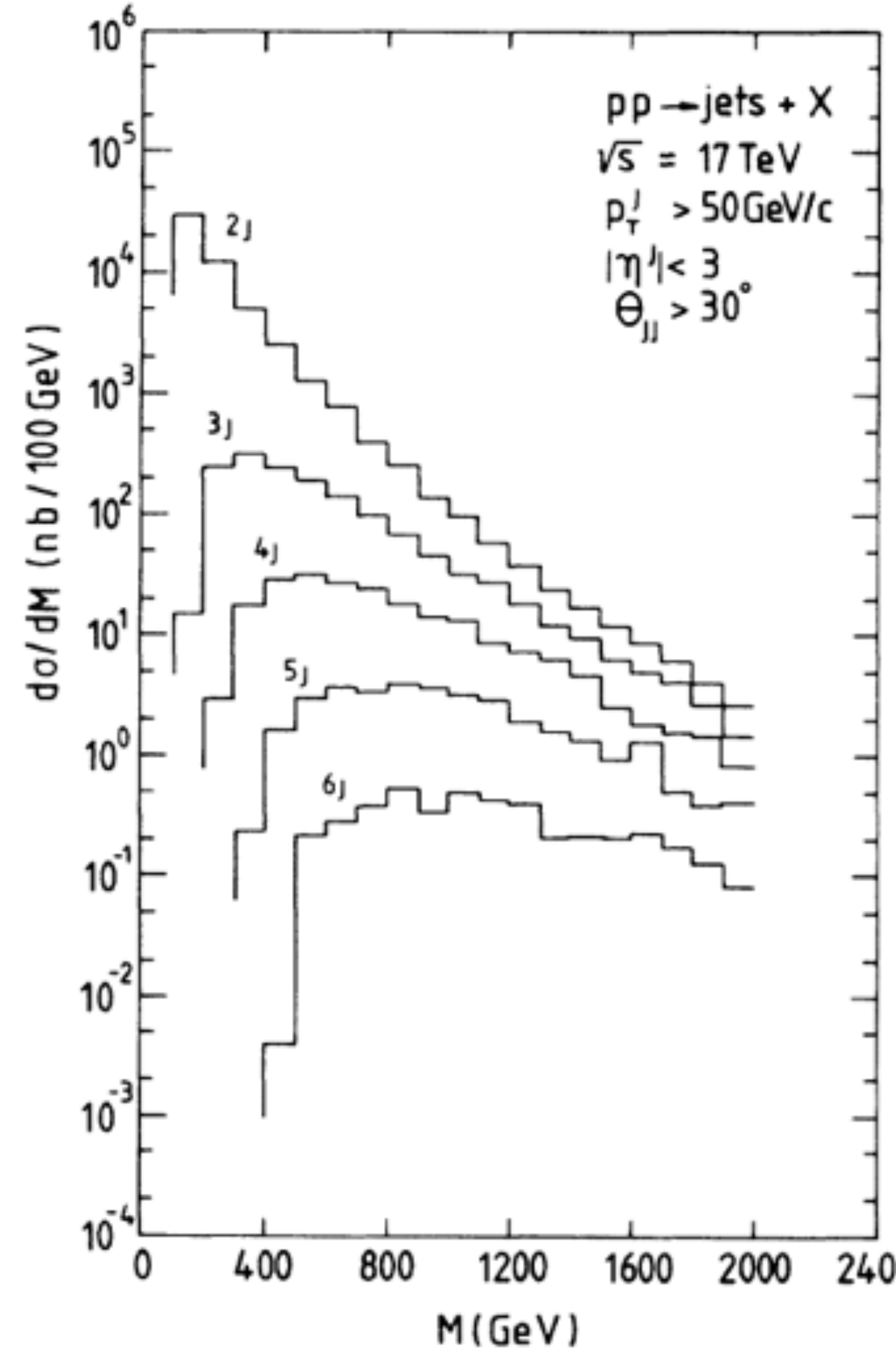
$$|\mathcal{M}(Q+;1-,2+,\ldots,n+;P-)|^2 = 2^{2-n} g^{2n} N^{n-1}(N^2-1)\frac{(P\cdot K_1)^3(Q\cdot K_1)}{(P\cdot Q)}$$

$$\times \sum_{P(1,\ldots,n)} \frac{1}{(Q\cdot K_1)(K_1\cdot K_2)\ldots(K_n\cdot P)} .$$

# Exact and Approximate multi-jet cross-sections

- Phenomenological significance?




$$f_{\rm eff}(x,Q^2)=f_g(x,Q^2)+\tfrac{4}{9}\sum_q\left[f_q(x,Q^2)+f_{\bar q}(x,Q^2)\right]$$

Multi - Jet Cross-sections in Hadronic Collisions, Z. Kunszt , W.J. Stirling . May 1987

Approximate Multi - Jet Cross-Sections in QCD, M. L. Mangano, S. J. Parke. Aug 1988.

Exact and Approximate Expressions for Multi - Gluon Scattering, F. A. Berends, W.T. Giele, H. Kuijf. May 1989

# Review on “multi-parton amplitudes in gauge theories”
# M. L. Mangano, S. J. Parke, Phys.Rept. 200 (1991)

- Parke-Taylor amplitude vs. “MHV amplitude”

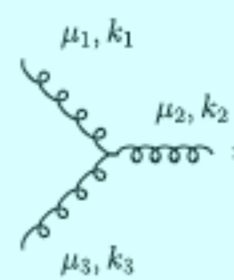
For purely gluonic processes, the MHV amplitudes are given by

$$M(g_1^-, g_2^-, g_3^+, \ldots, g_n^+) = i g^{n-2} \langle 12\rangle^4 \sum_{\{1,2,\ldots\}'} tr(\lambda_1 \lambda_2 \cdots \lambda_n) \frac{1}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

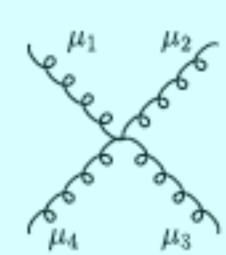
At the leading order in $N$ the square of these gluonic matrix elements, summed over colors, and over all the MHV configurations, gives the so called Parke and Taylor Amplitudes [80]:

$$|M(g_1, \ldots, g_n)|^2 = 2 g_s^{2n-4} N^{n-2}(N^2-1) \sum_{i>j} s_{ij}^4 \sum_{\{1,2,\ldots,n\}'} \frac{1}{s_{12} s_{23} s_{34} \ldots s_{n1}}.$$

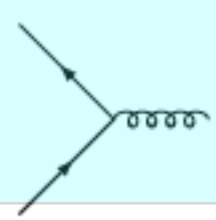
- Formulates color ordered Feynman rules for direct calculation of color ordered amplitudes



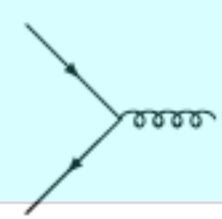

$$= i\frac{g_s}{\sqrt{2}}\left[(k_1-k_2)_{\mu_3}\, g_{\mu_1\mu_2} + (k_2-k_3)_{\mu_1}\, g_{\mu_2\mu_3} + (k_3-k_1)_{\mu_2}\, g_{\mu_3\mu_1}\right]$$




$$i\frac{g_s^2}{2}\left[2g_{\mu_1\mu_3}\, g_{\mu_2\mu_4} - g_{\mu_1\mu_4}\, g_{\mu_2\mu_3} - g_{\mu_1\mu_2}\, g_{\mu_3\mu_4}\right]$$



$$= i\frac{g_s}{\sqrt{2}}\gamma_\mu$$



$$= -i\frac{g_s}{\sqrt{2}}\gamma_\mu$$

- Excellent summary of tree level results for multi-jet physics up to year 1991

# MHV amplitudes in the period (1991-2003)

- Experimental: discovery of the top quark by CDF and D0

- 2 to 3 one loop amplitudes with helicity method

- Simplicity of Parke-Taylor amplitude are related to integrable models (Bardee

This simple structure is presumably related to the integrability properties of the self-dual theory. Can the methods developed to study integrable systems be applied to the construction of these correlation functions? It may be possible to develop a natural

- MHV amplitudes in the Regge limit (Del Duca)

- N=4 SYM, collinear limit, unitarity methods (Bern,Dunbar,Dixon,Kosower)

- Limited number NLO cross-section

  MCFM (Ellis, Campbell) , 3jet( Z. Nagy), VV'(Dixon,ZK,Signer) …

- ABDK conjecture (Anastasiou,Bern,Dixon,Kosower

The collinear factorization properties of two-loop scattering amplitudes in dimensionally-regulated N = 4 super-Yang-Mills theory suggest that, in the planar ('t Hooft) limit, higher-loop contributions can be expressed entirely in terms of one-loop amplitudes.

# Top discovery and the VECBOS generator

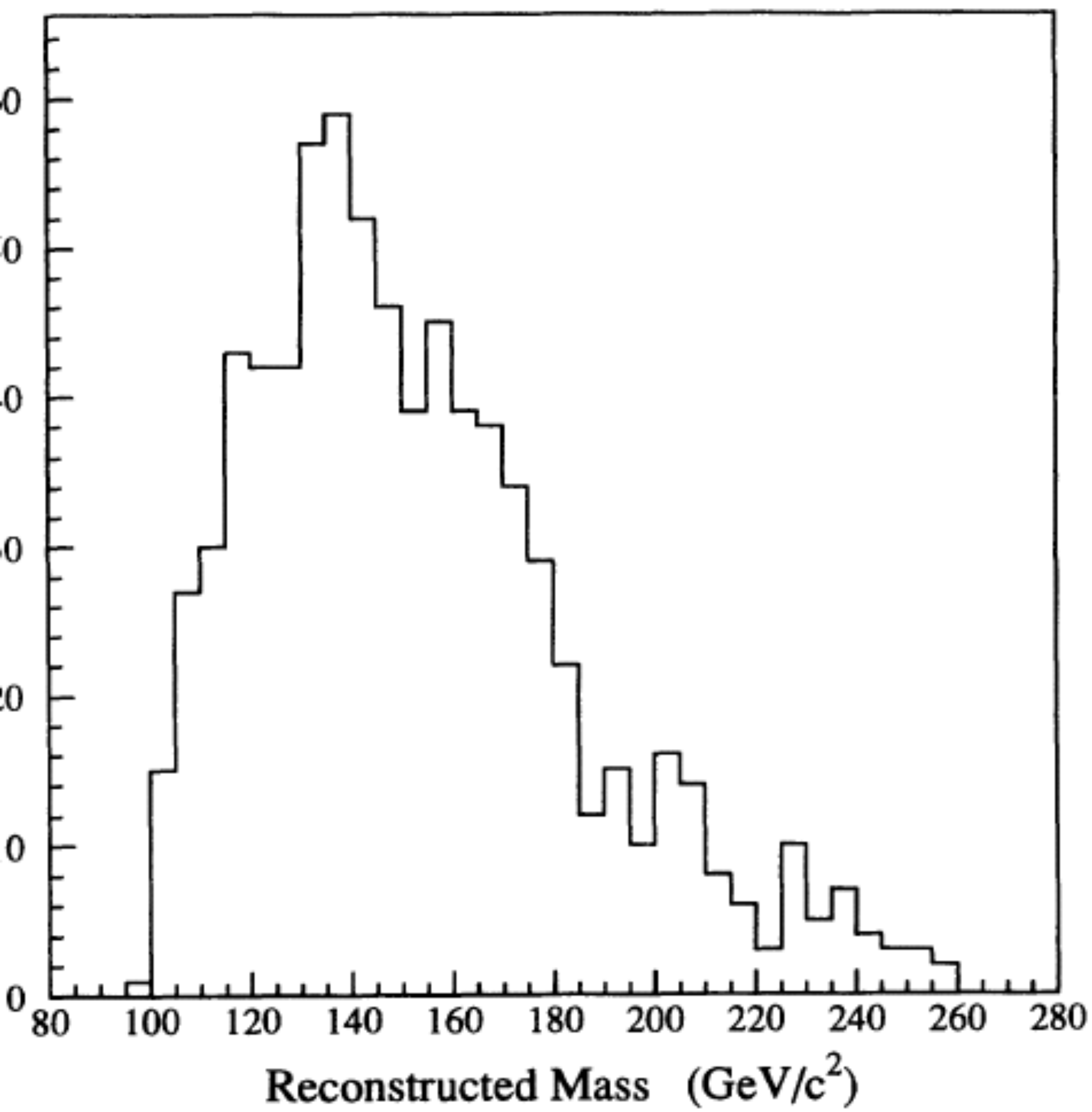
## Top production signals and irreducible background

VECBOS, tree level parton level generator for W+1,2,3,4 jets

On the production of a W and jets at hadron colliders (LO)
Berends, Giele,Kuijf, Tausk . 1990.

The Total Cross-Section for the Production of Heavy Quarks in Hadronic Collisions,. Nason, Dawson, Ellis, 1987, (NLO) and Benaaker et.al. (1988P) (traditional calculation)

# CDF: top can not be seen in the W+ multijet mass distribution




61. Reconstructed mass distribution for $W$+multijet Carlo events.

TABLE XVI. Comparison of $W$+jet(s) yields with expecta-ions from the VECBOS Monte Carlo program. The first uncer-ainty on the VECBOS prediction is due to Monte Carlo statistics, he second is due to jet energy scale and lepton identification fficiency uncertainties, the third is due to the luminosity nor-nalization. The VECBOS predictions include the $W \to \tau\nu$ contri-ution. The data have not been corrected for backgrounds, vhich are discussed in the text.

| et multiplicity | Data | VECBOS $(Q^2 = \langle P_T \rangle^2)$ |
|---|---|---|
| jet | 1713 | $1571 \pm 82^{+267}_{-204} \pm 55$ |
| jets | 281 | $267 \pm 20^{+77}_{-53} \pm 9$ |
| jets | 43 | $39 \pm 3^{+11}_{-9} \pm 2$ |
| ≥4 jets | 9 | $7 \pm 1^{+3}_{-2} \pm 0.2$ |

VECBOS was an important tool in top search

# Jet production at NLO

Calculation of Event Shape Parameters in e+ e- Annihilation
Ellis, Ross, Terrano. (1980). (subtraction method, no jets)

QCD Radiative Corrections to Parton Parton Scattering
K. Ellis, Sexton (1985). (traditional method, no jets)

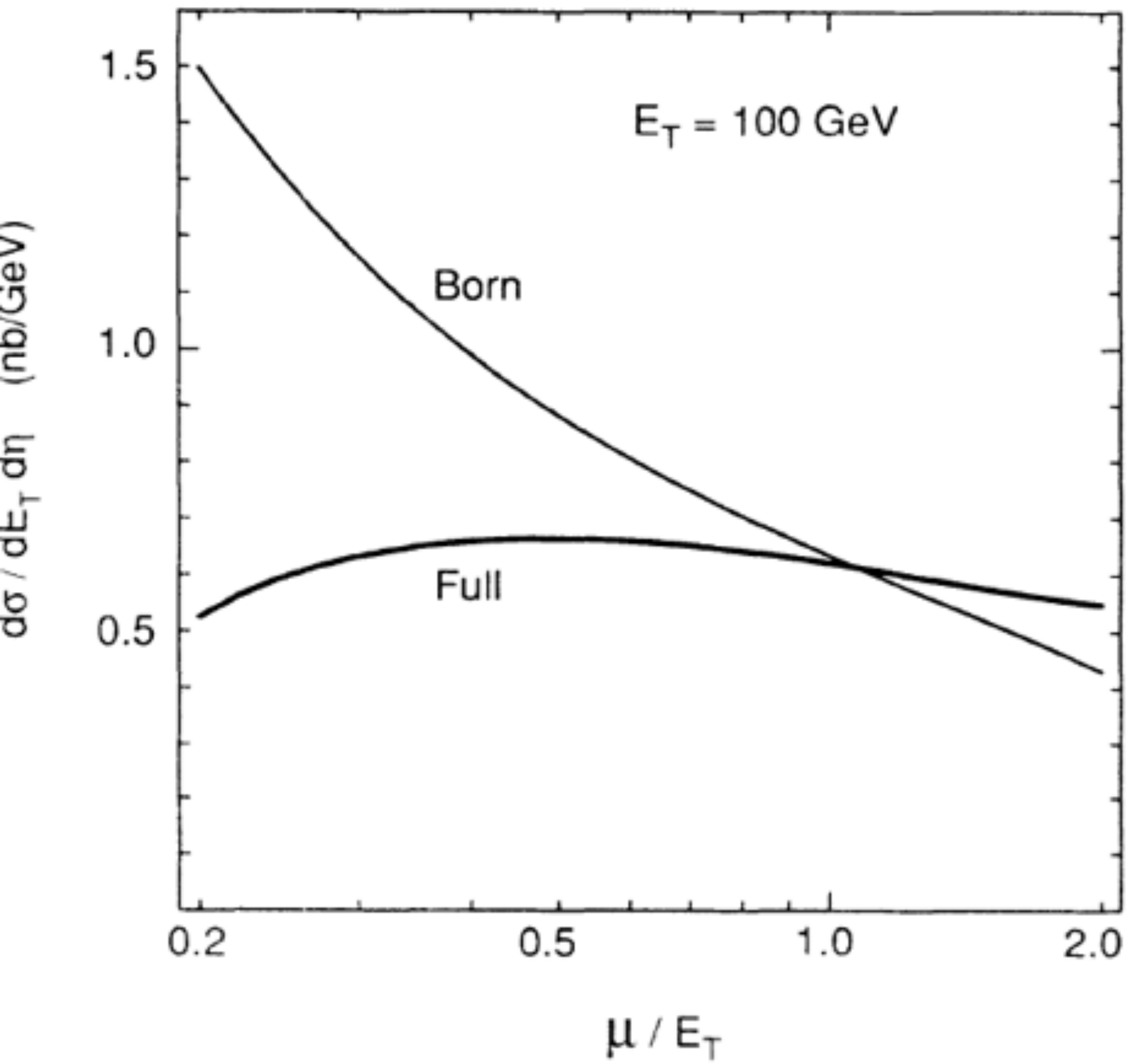
The One Jet Inclusive Cross-section at Order $\alpha_s^3$ Quarks and Gluons
S. Ellis, ZK, Soper (1990), (subtraction method, jets)

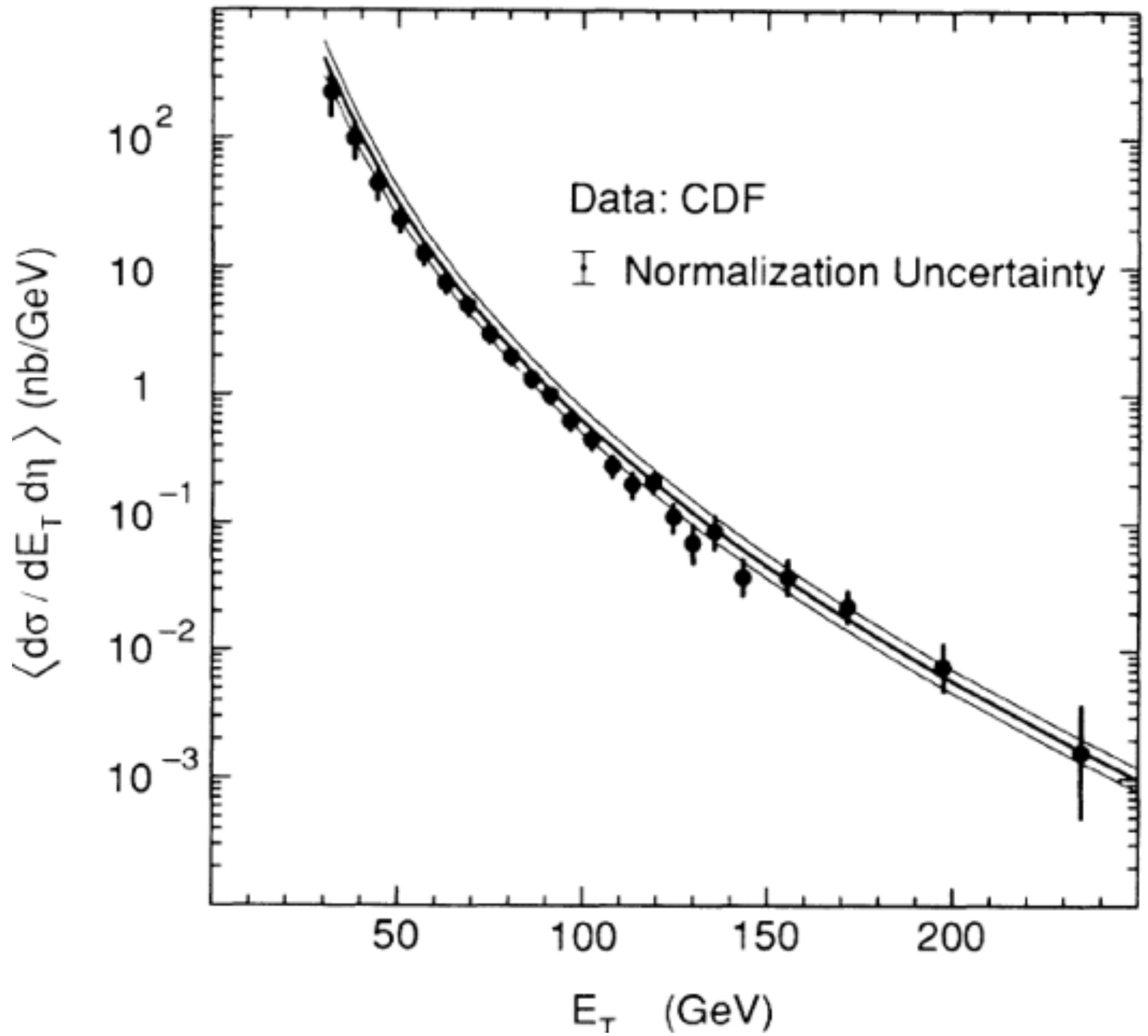
Higher order corrections to jet cross-sections in e+ e- annihilation
Giele, Glover (1991), (any number of gluon in the final state (universal structure of the infrared singularities)

Calculation of jet cross-sections in hadron collisions at order alpha-s**3
ZK, Soper, (1992) (detailed study of the infrared singularities)

Higher order corrections to jet cross-sections in hadron colliders
Giele, Glover, Kosower (1993).

# Inclusive jet production in NLO (EKS,CDF



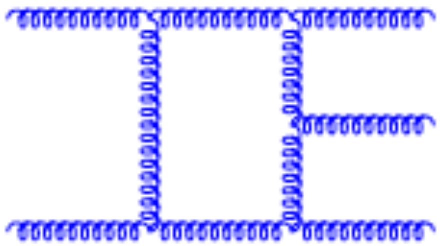




Get the theory prediction for three jet production !

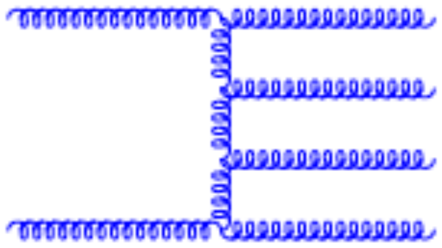
# Three jet production $d\sigma_{\mathrm{NLO}}$

one-loop $2 \to 3$ amplitudes: 5g, 2q3g (BDK,1993/1995), 4q1g (ZK,Signer,1994)

string based and helicity based method, SUSY, soft and collinear infrared poles, FHD regularization, primitive amplitudes



tree level $2 \to 4$ amplitudes (1985)



universal process independent method for cancelling the

dipole subtraction — Catani Seymour (1996)
residue subtraction — FKS (1995), Giele, Glover, Kosower (with slicing)
antenna subtraction — Kosower(1997), Campbell,Cullen,Gover,…
automated subtraction — Sherpa, Helac, Madipol,MadFKS,..

Jet definition and recombination algorithms

numerical implementation NLOJET++
Three jet cross-sections in hadron hadron collisions at next-to-leading order
Z. Nagy. 2001.

# MHV one-loop 5g amplitudes for (+++++,-++++,-+-++)

Z. Bern, L. J. Dixon, D. A. Kosower, 1993.

$$\mathcal{A}_n\left(\{k,\lambda_i,a_i\}\right) = \sum_J n_J \sum_{c=1}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;c}} \mathrm{Gr}_{n;c}\left(\sigma\right)\, A_{n;c}^{[J]}(\sigma)$$

$$\mathrm{Gr}_{n;1}(1) = N\,\mathrm{Tr}\left(T^{a_1}\cdots T^{a_n}\right),\ \mathrm{Gr}_{n;c}(1) = \mathrm{Tr}\left(T^{a_1}\cdots T^{a_{c-1}}\right)\mathrm{Tr}\left(T^{a_c}\cdots T^{a_n}\right)$$

For amplitudes +++++,-++++ $A_{n;c}^{[1]} = -A_{n;c}^{[1/2]} = A_{n;c}^{[0]}$ (supersymmetry)

$$A_{5;1}^{[1]}\left(1^+,2^+,3^+,4^+,5^+\right) = \frac{i}{96\pi^2}\frac{s_{12}s_{23}+s_{23}s_{34}+s_{34}s_{45}+s_{45}s_{51}+s_{51}s_{12}+\varepsilon(1,2,3,4)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle},$$

$$A_{5;1}^{[1]}\left(1^-,2^+,3^+,4^+,5^+\right) = \frac{i}{48\pi^2}\frac{1}{[1\,2]\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]}\Bigg[(s_{23}+s_{34}+s_{45})[2\,5]^2 - [2\,4]\,\langle 4\,3\rangle\,[3\,5]\,[2\,5] - \frac{[1\,2]\,[1\,5]}{\langle 1\,2\rangle\,\langle 1\,5\rangle}\left(\langle 1\,2\rangle^2\langle 1\,3\rangle^2\frac{[2\,3]}{\langle 2\,3\rangle} + \langle 1\,3\rangle^2\langle 1\,4\rangle^2\frac{[3\,4]}{\langle 3\,4\rangle} + \langle 1\,4\rangle^2\langle 1\,5\rangle^2\frac{[4\,5]}{\langle 4\,5\rangle}\right)\Bigg].$$

$$A_{5;1}^{[0]} = c_\Gamma\left(V^s A_5^{\mathrm{tree}} + iF^s\right),$$

$$A_{5;1}^{[1/2]} = -c_\Gamma\left((V^f+V^s)A_5^{\mathrm{tree}} + i(F^f+F^s)\right),$$

$$A_{5;1}^{[1]} = c_\Gamma\left((V^g+4V^f+V^s)A_5^{\mathrm{tree}} + i(4F^f+F^s)\right)$$

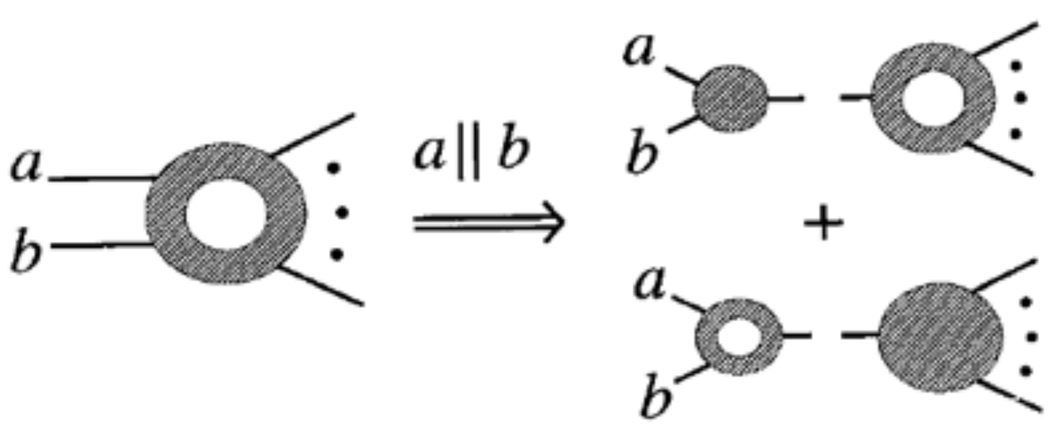
Bern, Dixon, Kosower from string theory to amplitudes (1992/1993)

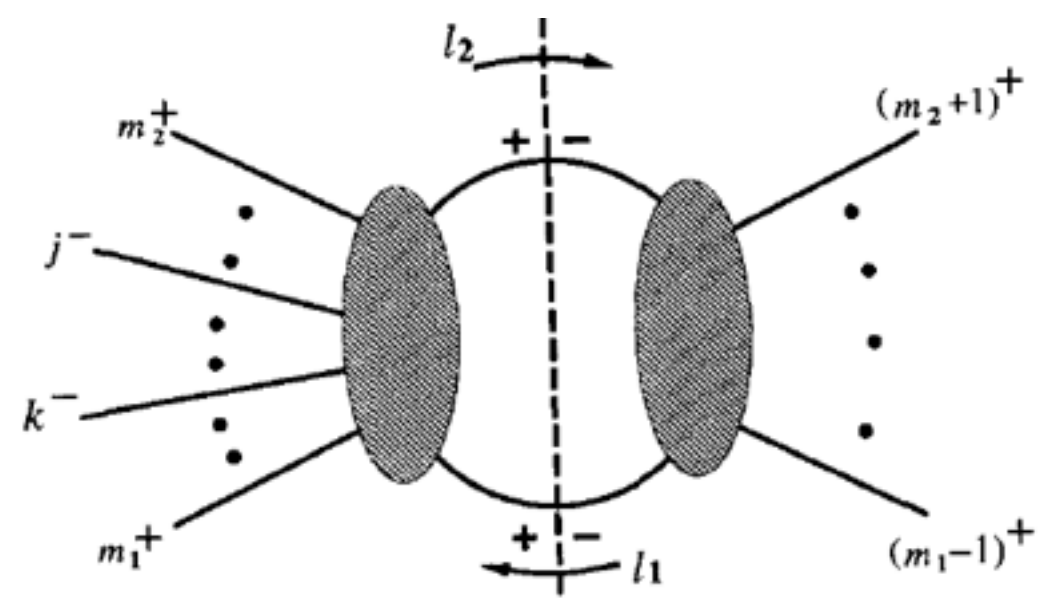
# MHV amplitudes for n-gluons in N=4 SYM theory

## Collinear limit and unitarity

One loop n point gauge theory amplitudes, unitarity, collinear limits and SUSY
Z. Bern, L. J. Dixon, D. C. Dunbar, D. A. Kosower Mar 7, 1994.


Fig. 1. To obtain all-$n$ expressions we impose a variety of constraints summarized here.

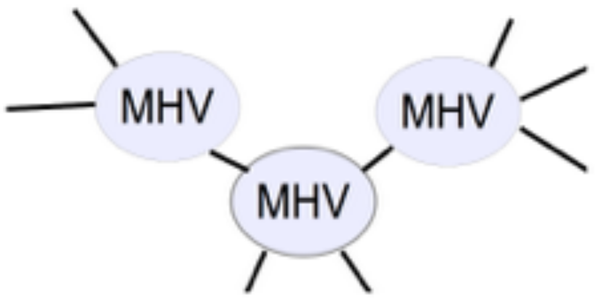


# MHV amplitudes in the period (2004-2015)

On-shell method for multi parton scattering amplitudes and complex kinematics

From twistor string theory to amplitudes (E.Witten(2003), CSW rules (Cachazo, Srvcek, Witten (2004))




Complex external on-shell momenta, a series of papers by
Britto, Cachazo,Feng in 2004/2005
BCFW recursion relations for tree amplitudes

Unitary cut method

$d\sigma_{\mathrm{NLO}}$ fully automated

# Three point amplitudes with complex kinematic

Witten

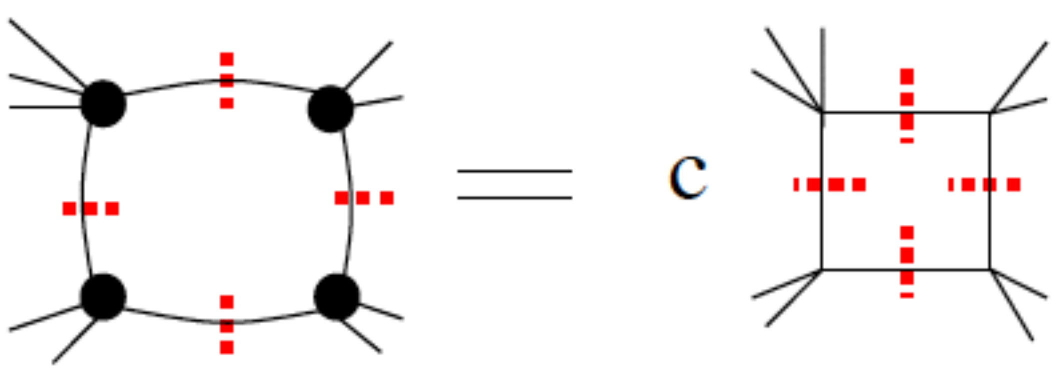
$$(\not{k}_i)_{\alpha\dot{\alpha}} \equiv k_i^{\mu}(\sigma_\mu)_{\alpha\dot{\alpha}} = u_+(k_i)\overline{u_+(k_i)} = (\lambda_i)_\alpha(\tilde{\lambda}_i)_{\dot{\alpha}}$$

$$A_3^{\text{tree}}(1^-,2^-,3^+) = i\,\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,1\rangle} \qquad A_3^{\text{tree}}(1^+,2^+,3^-) = -i\,\frac{[1\,2]^4}{[1\,2]\,[2\,3]\,[3\,1]}$$

$$\tilde{\lambda}_1^{\dot{\alpha}} \propto \tilde{\lambda}_2^{\dot{\alpha}} \propto \tilde{\lambda}_3^{\dot{\alpha}} \qquad \lambda_1^{\alpha} \propto \lambda_2^{\alpha} \propto \lambda_3^{\alpha}$$

## Complex momenta allow a 4-cut solutions

Britto, Cachazo, Feng




$c = A_1\, A_2\, A_3\, A_4$

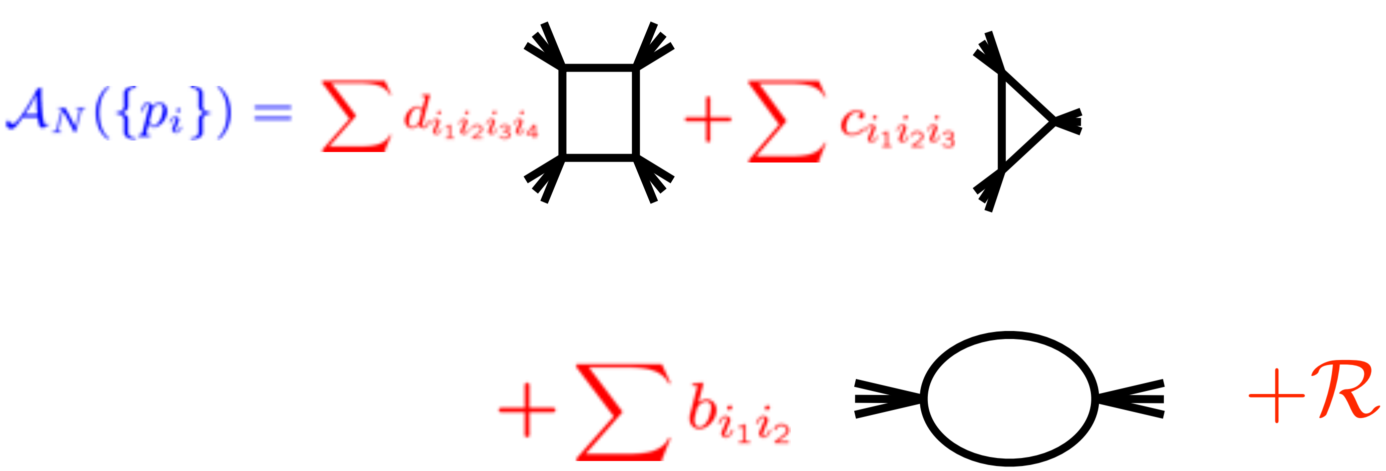
# On-shell methods of calculating amplitudes in QCD and N=4 sYM at large N

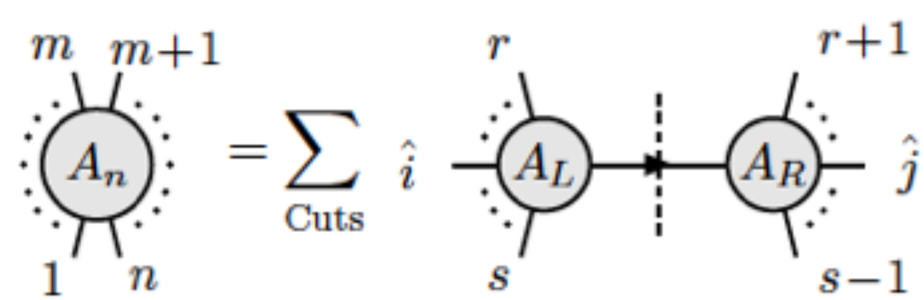
Helicity method.supersymmetry
unitarity cuts (BDK), OPP, cuts in D-dim.
soft gluon limits of amplitudes
Anomalous dimensions at NNLO
Regge limit
Automated NLO codes for phenomenology (MCFM, BH, Sherpa, Powheg)

Complexification of external moment with twistors
CSW and BCFW recursion relations
BDS Ansatz;
Amplitudes and Wilson loops
Dual conformal invariance,
Yangian symmetry, Integrability
Relation to N=8 supergratvity

QCD

N=4 sYM



# Basic ideas of the unitarity method: one-loop amplitudes from tree amplitudes + scalar integral functions

- First application of unitarity method (BDDK magic): analytic matrix elements for $q\bar{q}ggV,\ q\bar{q}Q\overline{Q}V$ used in MCFM

Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, Nucl. Phys. **B425**, 217 (1994); **B435**, 59 (1995).

- Decompose the amplitude in terms of basic set of scalar integral functions and read out the coefficients using unitarity cuts (‘98) (BDK)

- Use generalized cuts, read out the coefficients in terms of tree amplitudes at cut-momenta (complex) BCF/BDK(’05)

- Consider the integrand, the amplitude is parametric integral over the loop momentum Ossola,Papadopoulos,Pittau(’06), (Ellis,Giele,ZK(2007)), allows also fast numerical implementation

  - Unitarity cut method in D-dimension, rational parts. Bern,Morgan(1996); Anastasiou,Britto,Feng, Mastrolia(2006); Giele, ZK,Melnikov.(2007), Badger (2008)

# NLO amplitudes for the scattering of up to 20 gluon

CPU time: N^9, 7^N, (N!!)^2

Giele and Zanderighi

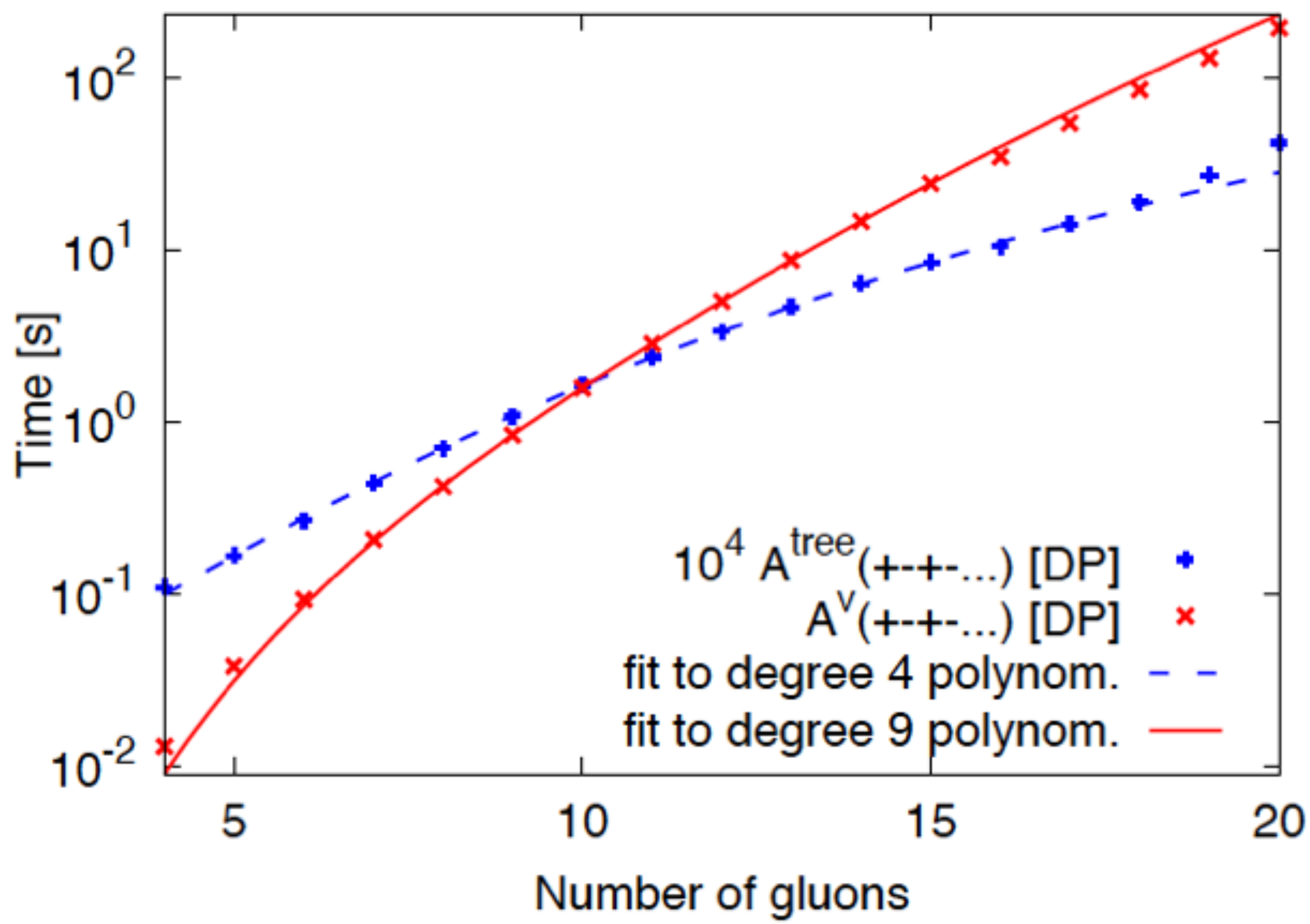


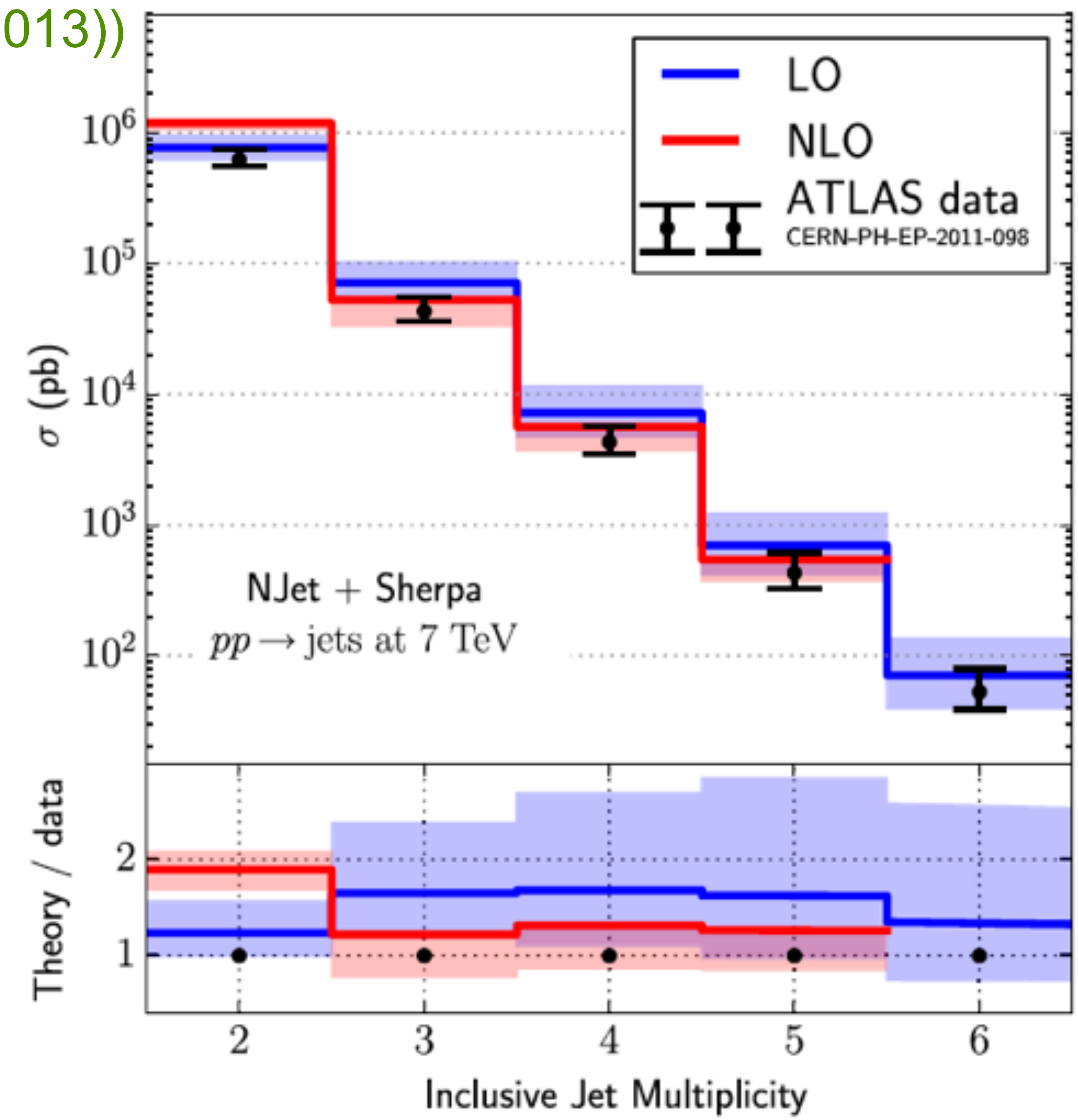

**Figure 3:** Time in seconds needed to compute tree (blue, dashed) and one-loop (red, solid) ordered amplitudes with gluons of alternating helicity signs, $A_N^{[1]}(+-+-+...)$, as a function of the number of external gluons ranging between 4 to 20 using a single 2.33 GHz Xeon processor.

# Automated calculation of $\mathrm{d}\sigma_{\mathrm{NLO}}$

HELAC/CutTools, Rocket, BlackHat+SHERPA,GoSam+Sherpa, Njet+SHERPA, MadGraph5_aMC@NLO

# Exact multi-jet cross-sections at NLO

Computation of multi-leg amplitudes with NJet (Badger, Biedermann , Uwer , Yundin (2013))




Fast jet reconstruction algorithm

$$\mathcal{O}(N^3) \to \mathcal{O}(N \ln(N))$$

FastJet User Manual
(Cacciari, Salam, Soyez (2011))

NLO multi-jet: A Practical Seedless Infrared-Safe Cone jet algorithm
(Salam, Soyez . (2007))

# $\mathrm{d}\sigma_{\mathrm{NNLO}}$ (2004-2015)

Two loop matrix elements

Methods for cancellation of infrared poles in $\mathrm{d}\sigma_{\mathrm{NNLO}}$

Bottleneck of NNLO predictions: two loop amplitudes

Two obstruction:

reduction of two loop multi-leg amplitudes

analytic and/or fast numerical evaluation of all elements of some Master Integral Basis

# Two loop amplitudes

## Double box scalar integral, IBP relations, Laporta algorithm

V.A. Smirnov, Phys. Lett. B **460**, 397 (1999).
J.B. Tausk, Phys. Lett. B **469**, 225 (1999).
V.A. Smirnov and O.L. Veretin, Nucl. Phys. **B566**, 469 (2000).
C. Anastasiou, T. Gehrmann, C. Oleari, E. Remiddi, and J.B. Tausk, Nucl. Phys. **B580**, 577 (2000).
T. Gehrmann and E. Remiddi, Nucl. Phys. **B580**, 485 (2000).
C. Anastasiou, E.W.N. Glover, and C. Oleari, Nucl. Phys. **B565**, 445 (2000); **B575**, 416 (2000).
V.A. Smirnov, Phys. Lett. B **491**, 130 (2000); T. Gehrmann and E. Remiddi, Nucl. Phys. **B601**, 248 (2001); **B601**, 287 (2001);

S. Laporta, Int.J.Mod.Phys. A15 (2000) 5087-5159

# NNLO amplitudes

Known two loop matrix elements

$$2 \to 1 : q\bar{q} \to V, gg \to H, q\bar{q} \to HV$$

$$2 \to 2 : \text{all with 4 massless partons,}$$

Anastasiou, Glover, Oleari, Tejeda-Yeomans (2001), Bern, Freitas, Dixon (2002)

$$2 \to 2 : \text{all with 3 massless partons plus V or H}$$

Anastasiou, Glover, Oleari, Tejeda-Yeomans (2001), Bern, Freitas, Dixon (2002)

$$2 \to 2 : q\bar{q} \to VV, gg \to VV$$

Anastasiou, Glover, Oleari, Tejeda-Yeomans (2001), Bern, Freitas, Dixon (2002)

$$2 \to 2 : q\bar{q} \to t\bar{t}, gg \to t\bar{t} \text{ ( only numerically)}$$

Bernreuter, Czakon,Mitov (2012)

all using traditional methods

# Methods for cancellation of infrared poles in $d\sigma_{\mathrm{NNLO}}$

$$
\begin{aligned}
d\sigma^{RR,\mathrm{sub}} &= \int_{n+2} \left(d\sigma^{RR} - d\sigma^{RR,\mathrm{single}} - d\sigma^{RR,\mathrm{double}}\right) \\
d\sigma^{RV,\mathrm{sub}} &= \int_{n+1} \left(\left[d\sigma^{RV} + \int_1 d\sigma^{RR,\mathrm{single}}\right] - d\sigma^{RV,\mathrm{single}}\right) \\
d\sigma^{VV,\mathrm{fin}} &= \int_{n} \left(d\sigma^{VV} + \int_1 d\sigma^{RV,\mathrm{single}} + \int_2 d\sigma^{RR,\mathrm{double}}\right)
\end{aligned}
$$

- QT subtraction [Catani Grazzini; 2005] , not universal
- Antenna - Subtraction [Gehrmann, Glover et al., Weinzierl, 2005]
- Somogyi, Trócsányi (ST) subtraction (only for e+e-)
- Czakon, stripper
- Jettines [Boughezal, Melnikov, Petriello, 2011]
- ……..

# Precision top quark physics at the LHC

VECBOS $\rightarrow$ BLACKHAT

Multi-jet cross sections at NLO with BlackHat and Sherpa
C.F. Berger et.al. May 2009.

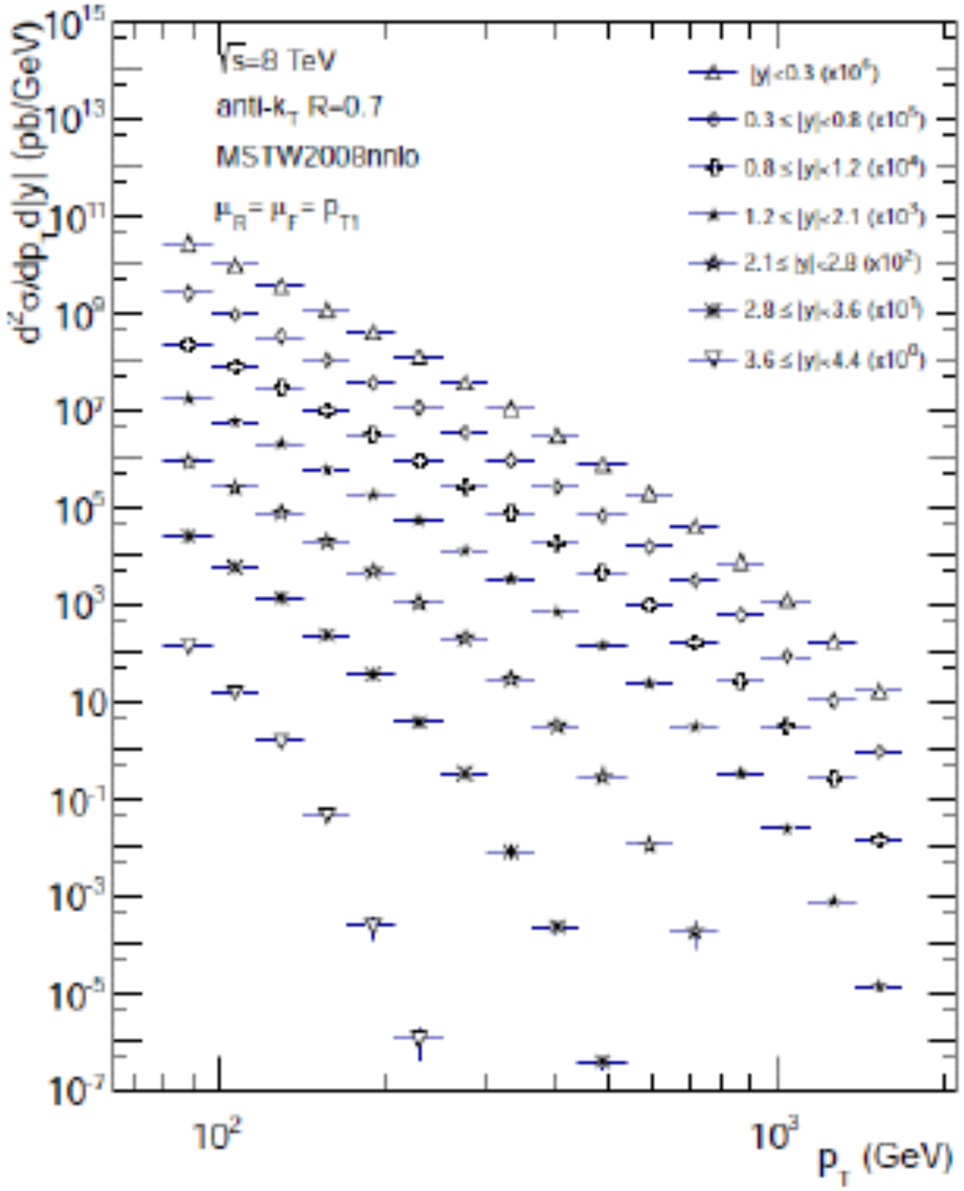
NLO QCD corrections to top quark pair production in association with one hard jet at hadron colliders, K. Melnikov, M. Schulze, Apr 2010.

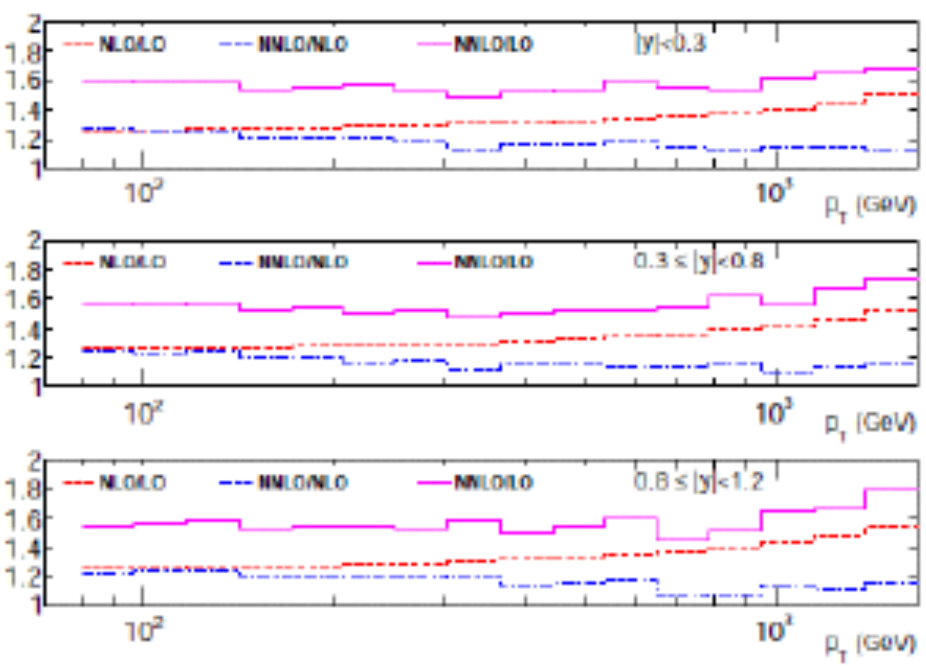
High-precision differential predictions for top-quark pairs at the LHC
M. Czakon, D. Heymes, A. Mitov. Nov 2, 2015. (NNLO)

# Di-jets production at NNLO

Currie, Gehrmann-Ridder, Glover, Pires *2014

- Double differential distribution R=0.7




double differential k-factors

- NNLO result varies between 25% to 12% with respect to the NLO cross section
- similar behaviour between the rapidity slices

# Unitarity method for two loop 5gluon all plus helicity amplitude

D-dimensional multi-loop integrand reduction method


P. Mastrolia and G. Ossola *JHEP* **1111** (2011) 014, [1107.6041].

S. Badger, H. Frellesvig, and Y. Zhang *JHEP* **1204** (2012) 055, [1202.2019].

Y. Zhang *JHEP* **1209** (2012) 042, [1205.5707].

P. Mastrolia, E. Mirabella, G. Ossola, and T. Peraro *Phys.Lett.* **B718** (2012) 173–177, [1205.7087].

S. Badger, H. Frellesvig, and Y. Zhang *JHEP* **1208** (2012) 065, [1207.2976].

P. Mastrolia, E. Mirabella, G. Ossola, and T. Peraro *Phys.Rev.* **D87** (2013) 085026, [1209.4319].

R. H. Kleiss, I. Malamos, C. G. Papadopoulos, and R. Verheyen *JHEP* **1212** (2012) 038, [1206.4180].

B. Feng and R. Huang *JHEP* **1302** (2013) 117, [1209.3747].


Complete Two-Loop, Five-Gluon Helicity Amplitude in Yang-Mills Theory

(Badger,Moguli,Ochirov, O' Connel, 2015)

Analytic form of the two-loop planar five-gluon all-plus-helicity amplitude in QCD

(Gehrmann , Henn, Lo Presti, Nov. 2015)

We compute the full set of planar master integrals relevant to five-point functions in massless QCD, and use these to derive an analytical expression for the two-loop five-gluon all-plus-helicity amplitude. A five point amplitude is calculated analytically in QCD the first time.

# Summary

- Spectacular technical advances in experimental high energy physics:
  i) precise measurements of multi-jet observables
  ii) precision of the theoretical predictions have to match the experimental one.

- Traditional decomposition of QCD scattering amplitudes in terms of Feynman diagrams:
  i) highly inefficient for describing amplitudes with many legs or/end loops order
  ii) new ideas to abandon partially or fully the diagrammatic approaches.

- The discovery of tree level MHV amplitudes triggered new revolutionary approaches :
  i) powerful use of helicity techniques, color decompositions, supersymmetry, recursion relations, on-shell unitarity, universal properties of collinear and soft limits, complexification of the external kinematics
  ii) full automation of the calculation of multi jet observables at NLO accuracy.

# Summary

- Expectations to the near and not to near future:
  - i) NNLO will be derived for all important benchmark processes
  - ii) Unitarity method for two loop NNLO multi-leg calculations with proper two loop master integral basis
  - iii) Automation of the NNLO calculations ?