A Curious Story of Quantum Gravity in the Ultraviolet

MHV@30
Fermilab
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Zvi Bern
UCLA

ZB, Carrasco, Johansson, arXiv:1004.0476
ZB, T. Dennen, S. Davies, V. Smirnov and A. Smirnov, arXiv:1309.2496
ZB, T. Dennen, S. Davies, arXiv:1409.3089; arXiv:1412.2441
ZB, C. Cheung, H.H. Chi, S. Davies, L. Dixon, J. Nohle. arXiv:1507.06118
ZB, S. Davies, J. Nohle, arXiv:1510.03448

Simplicity in Scattering Amplitudes

For the history, see other talks: Kunszt, Kosower, Hodges and others.
Here I will only talk about history directly relevant for the rest of my talk.

28 years ago David Kosower mentioned the "Parke-Taylor formula".

I said, "What's that?" (Words to be forgotten!)

David Kosower's response should be immortalized:

"Everyone needs to know the Parke-Taylor formula!"

David was right.
28 years later everyone does indeed know it!

MHV amplitude in spinor notation: Mangano, Parke and Xu (1988)

$$A(1^-,2^-,3^+,\ldots,n^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

MHV Amplitudes

(educated guess)

$$A(1^-,2^-,3^+,\ldots,n^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

It wasn't so obvious this formula would be important:

- **It's too special. Limited helicities.**
- **No masses.**
- **Not directly applicable to phenomenology.**
- **No obvious generalization to loops.**
- **Etc.**

But those of us who were young at the time did not know we were supposed to worry about the above problems.

Instead we could see:

- **Great beauty and simplicity!**
- **Huge potential for loops! A revolution waiting to happen!**

Why would we *not* want to work on generalizing this?!!
Instead of problems we saw opportunities!

See Kosower's talk

Simplicity at Loops

BDK 1993

Using string-based methods, with Lance Dixon, David Kosower we obtained the one-loop QCD five-gluon amplitude proving simplicity at loop level.

$$V^g = -\frac{1}{\epsilon^2}\sum_{j=1}^{5}\left(\frac{\mu^2}{-s_{j,j+1}}\right)^{\epsilon} + \sum_{j=1}^{5}\ln\left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}}\right)\ln\left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}}\right) + \frac{5}{6}\pi^2$$

***N* = 4 sYM**

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{23}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right)\right] - 2, \qquad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

$$F^f = -\frac{1}{2}\frac{\langle 1\,2\rangle^2\left(\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle + \langle 2\,4\rangle\,[4\,5]\,\langle 5\,1\rangle\right)}{\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\,\frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3}\frac{[3\,4]\,\langle 4\,1\rangle\,\langle 2\,4\rangle\,[4\,5]\left(\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle + \langle 2\,4\rangle\,[4\,5]\,\langle 5\,1\rangle\right)}{\langle 3\,4\rangle\,\langle 4\,5\rangle}\,\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{1}{3}F^f$$

$$-\frac{1}{3}\frac{\langle 3\,5\rangle\,[3\,5]^3}{[1\,2]\,[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]} + \frac{1}{3}\frac{\langle 1\,2\rangle\,[3\,5]^2}{[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]} + \frac{1}{6}\frac{\langle 1\,2\rangle\,[3\,4]\,\langle 4\,1\rangle\,\langle 2\,4\rangle\,[4\,5]}{s_{23}\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,s_{51}}$$

Certain loop-level helicity amplitudes are simple!

***N* = 4 sYM even simpler!**

MHV Gravity Amplitudes

Kawai-Lewellen-Tye relations (derived from string theory):

gravity

gauge theory

$$M_4^{\rm tree}(1,2,3,4) = -i s_{12} A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)\,,$$

$$M_5^{\rm tree}(1,2,3,4,5) = i s_{12} s_{34} A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5)$$

$$+ i s_{13} s_{24} A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5)$$

Etc.

(educated guess)

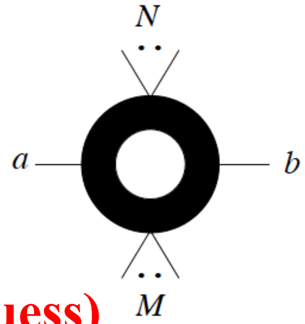
$$M_n^{\rm tree}(1^-,2^-,3^+,\ldots,n^+) = -i\,\langle 1\,2\rangle^8$$

$$\times \left[\frac{[1\,2]\,[n-2\;\;n-1]}{\langle 1\;\;n-1\rangle\; N(n)} \left(\prod_{i=1}^{n-3} \prod_{j=i+2}^{n-1} \langle i\,j\rangle \right) \prod_{l=3}^{n-3} \Big(-\langle n^-|\not{K}_{l+1,n-1}|l^-\rangle \Big) \; + \; \mathcal{P}(2,3,\ldots,n-2) \right]$$

Berends, Giele and Kuijf

MHV gravity tree amplitudes are simple!

MHV One-loop Gravity Amplitudes



ZB, Dixon, Rozowsky, Perelstein (1998)

Pure gravity one-loop identical helicity:

(educated guess)

$$M_n(1^+,2^+,\ldots,n^+) = -\frac{i\,(-1)^n}{(4\pi)^2\cdot 960} \sum_{\substack{1\le a<b\le n\\ M,N}} h(a,M,b)h(b,N,a)\,\mathrm{tr}^3[a\,M\,b\,N]$$

$$h(a,\{1,2,\ldots,n\},b) \equiv \frac{[1\,2]}{\langle 1\,2\rangle}\,\frac{\langle a^-|\not{K}_{1,2}|3^-\rangle\langle a^-|\not{K}_{1,3}|4^-\rangle\cdots\langle a^-|\not{K}_{1,n-1}|n^-\rangle}{\langle 2\,3\rangle\,\langle 3\,4\rangle\cdots\langle n-1,\,n\rangle\,\langle a\,1\rangle\,\langle a\,2\rangle\,\langle a\,3\rangle\cdots\langle a\,n\rangle\,\langle 1\,b\rangle\,\langle n\,b\rangle} + \mathcal{P}(2,3,\ldots,n),$$

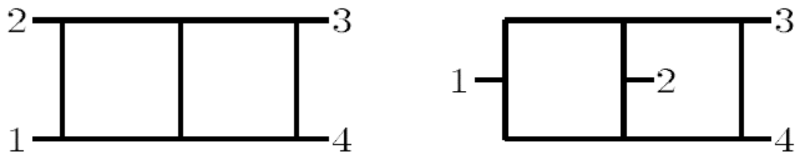
***N* = 8 supergravity MHV amplitude**

(educated guess)

$$M_n^{N=8}(1^-,2^-,3^+,\ldots,n^+) = \frac{(-1)^n}{8}\,\langle 1\,2\rangle^8 \sum_{\substack{1\le a<b\le n\\ M,N}} h(a,M,b)h(b,N,a)\,\mathrm{tr}^2[a\,M\,b\,N]\,\mathcal{I}_4^{aMbN}$$

One-loop MHV gravity amplitudes are simple!

Multi Loop Integrands



ZB, Yan, Rozowsky (1997);
ZB, Dixon, Rozowsky, Perelstein (1998)

Multiloop integrands in *N* = 4 sYM susy are simple!

$$\mathcal{A}_4^{\text{2-loop}}(1,2,3,4) = -g^6 s_{12}s_{23}\, A_4^{\text{tree}}(1,2,3,4)\Big(C^{\rm P}_{1234}\, s_{12}\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{23}) + C^{\rm P}_{3421}\, s_{12}\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{24}) + C^{\rm NP}_{1234}\, s_{12}\,\mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{23}) + C^{\rm NP}_{3421}\, s_{12}\,\mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{24}) + \text{cyclic}\Big)\,,$$

Scalar double boxes ↗

Simplicity remains for integrated expressions! **See Lance's talk**

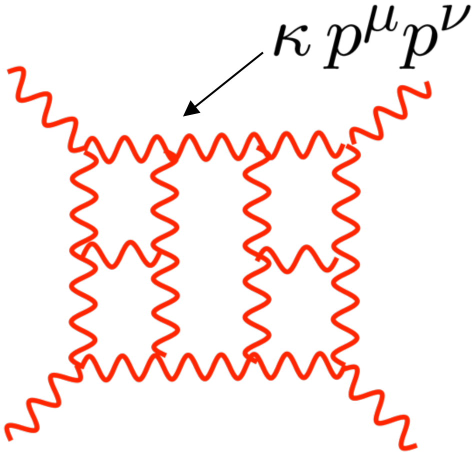
N = 8 supergravity integrands just as simple!

$$\mathcal{M}_4^{\text{2-loop}}(1,2,3,4) = -i\Big(\frac{\kappa}{2}\Big)^6 [s_{12}s_{23}\, A_4^{\text{tree}}(1,2,3,4)]^2 \Big(s_{12}^2\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{23}) + s_{12}^2\, \mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{24}) + s_{12}^2\, \mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{23}) + s_{12}^2\, \mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{24}) \quad + \text{cyclic}\Big)$$

Simplicity of gravity integrands is key for rest of the talk.
The most powerful means available for studying UV in gravity!

Ultraviolet Behavior of Gravity

$\kappa = \sqrt{32\pi G_N}$ ← **Dimensionful coupling**



Einstein gravity:
$$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(\kappa p_j^\mu p_j^\nu)}{\text{propagators}}$$

Gauge theory:
$$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g p_j^\nu)}{\text{propagators}}$$

Extra powers of loop momenta in numerator:
Integrals are badly behaved in the UV.

Origin of simplistic statement that all point-like theories of gravity must be ultraviolet divergent.

Are we sure there must be divergence?
Cancellations between pieces?

Test case: $N = 8$ Supergravity

The best theories to look at are supersymmetric theories.

Supersymmetry relates bosons (forces) and fermions (matter)

We first consider $N = 8$ supergravity.

Einstein gravity + 254 other physical states

Reasons to focus on $N \geq 4$ supergravity:

- **With more supersymmetry expect better UV properties.**
- **High symmetry implies technical simplicity.**

In the late 70's and early 80's supergravity was seen as the primary means for unifying gravity with other forces.

Ferrara, Freedman, van Nieuwenhuizen

UV Finiteness of $N = 8$ Supergravity?

If $N = 8$ supergravity is perturbatively finite it would imply a new symmetry or non-trivial dynamical mechanism.

Such a mechanism would have a fundamental impact on our understanding of gravity.

Of course, perturbative finiteness is not the only issue for consistent gravity:

- Nonperturbative completions?
- High-energy behavior?
- Realistic models?

Here we are trying to answer a simple question:

Is $N = 8$ supergravity ultraviolet finite to all order of perturbation theory? Yes, or no?

Opinions from the 80's

> If certain patterns that emerge should persist in the higher orders of perturbation theory, then … $N = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops.
>
> Green, Schwarz, Brink, (1982)

> It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions… The final word on these issues may have to await further explicit calculations.

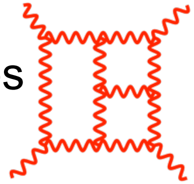
Marcus, Sagnotti (1985)

The idea that *all* supergravity theories diverge has been accepted wisdom for over 25 years, with a only a handful of contrarian voices.

Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

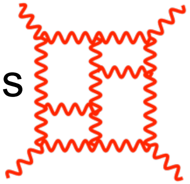
3 loops



$\sim 10^{20}$ TERMS

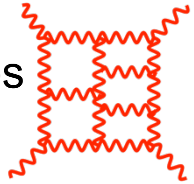
No surprise it has never been calculated via Feynman diagrams.

4 loops



$\sim 10^{26}$ TERMS

5 loops



$\sim 10^{31}$ TERMS

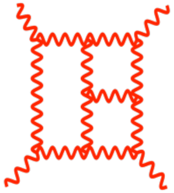
More terms than atoms in your brain!

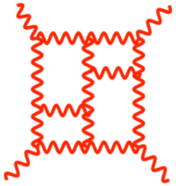
- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

With Modern Ideas

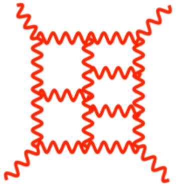
Z B, Carrasco, Dixon, Johansson, Roiban

For *N* = 8 supergravity.

3 loops  ~ 10 TERMS

4 loops  $\sim 10^2$ TERMS

Much more manageable!

5 loops  $\sim 10^4$ TERMS (Not yet done—1000 diagrams)

We now have the ability to settle the 35 year debate and determine the true UV behavior gravity theories.

Where is First Potential $D = 4$ UV Divergence?

3 loops **$N = 8$**	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops **$N = 8$**	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops **$N = 8$**	Howe and Stelle (2003)	✗
7 loops **$N = 8$**	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
3 loops **$N = 4$**	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops **$N = 5$**	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops **$N = 4$**	Vanhove and Tourkine (2012)	✓
9 loops **$N = 8$**	Bekovits, Green, Russo and Vanhove (2009)	✗

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

Don't bet on this now!

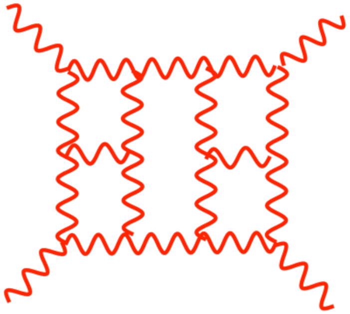
"Enhanced cancellations"

Weird structure.
Anomaly
behind divergence.

retracted

- **Conventional wisdom: divergence are expected at some high loop order.**
- **So far, *every* specific prediction of divergences in pure supergravity has either been wrong or missed crucial details.**

New Structures?



Might there be a new unaccounted structure in gravity theories that suggests the UV might be is tamer than conventional arguments suggest?

Yes!

Duality Between Color and Kinematics

Consider five-point tree amplitude:

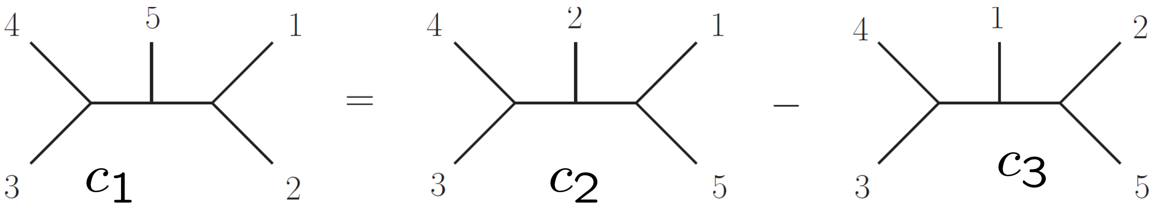
ZB, Carrasco, Johansson (BCJ)

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_i^2}$$

color factor

kinematic numerator factor

Feynman propagators



$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

See talks from Carrasco and Johansson

$$\boxed{c_1 + c_2 + c_3 = 0 \ \Leftrightarrow\ n_1 + n_2 + n_3 = 0}$$

Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.

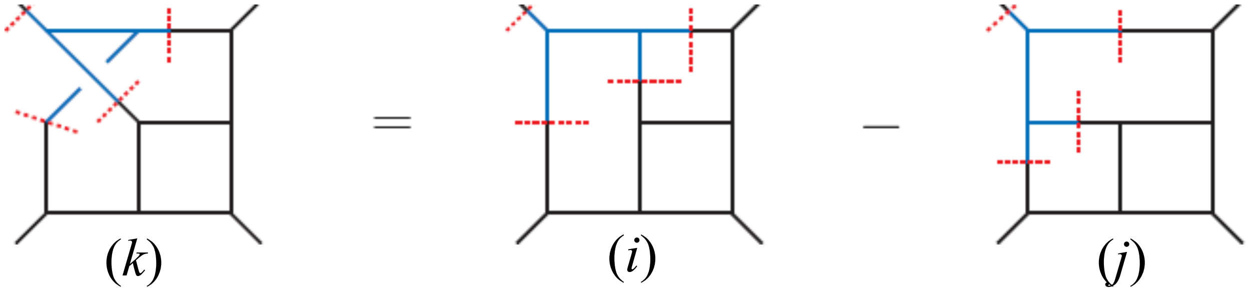
Progress on unraveling relations.

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer; Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White, etc.

BCJ

Gravity loop integrands are free!

Ideas conjectured to generalize to loops:



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory → gravity theory

simply take

color factor → kinematic numerator

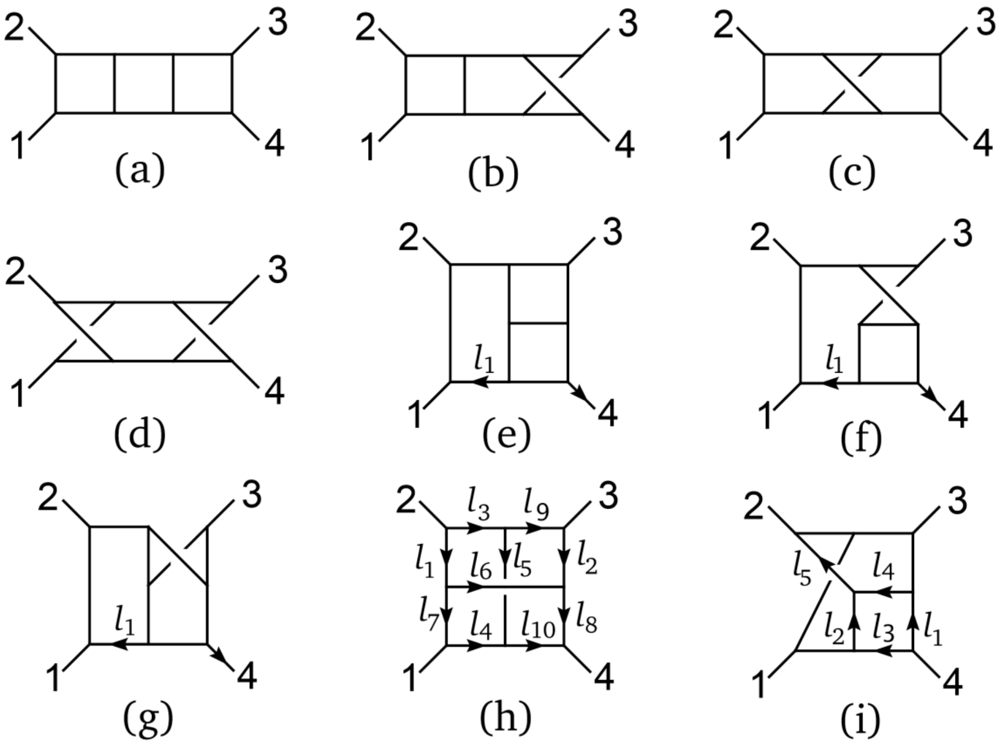
$$c_k \longrightarrow n_k$$

Gravity loop integrands follow from gauge theory!

$N = 8$ Supergravity Three-Loop Result

ZB, Carrasco, Dixon, Johansson, Kosower, Roiban; hep-th/0702112
ZB, Carrasco, Dixon, Johansson, Roiban arXiv:0808.4112 [hep-th]

Obtained via on-shell unitarity method:



Three-loop is not only ultraviolet finite it is "superfinite"—cancellations beyond those needed for finiteness in four dimensions.

UV finite for $D < 6$

More Recent Opinion

In 2009 Bossard, Howe and Stelle had a careful look at the question of how much supersymmetry can tame UV divergences.

In particular … suggest that maximal supergravity is likely to diverge at four loops in $D = 5$ and at five loops in $D = 4$ …

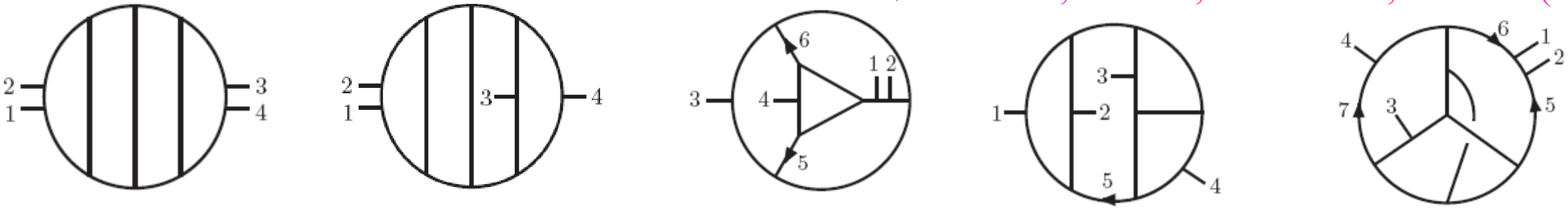
Bossard, Howe, Stelle (2009)

Bottles of wine were at stake!

We had tools to collect the wine.

$N = 8$ Supergravity Four-Loop Calculation

ZB, Carrasco, Dixon, Johansson, Roiban (2010)



50 distinct planar and non-planar diagrammatic topologies

UV finite for $D = 4$ and 5

actually finite for $D < 11/2$

Very very finite.

A very nice Barolo!

A New Consensus from Supergravity Experts

More recent papers argue that trouble starts at 5 loops and by 7 loops we have valid potential UV counterterm in $D = 4$, accounting for all known symmetries.

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Kallosh; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

All previous calculations explained and divergences predicted.

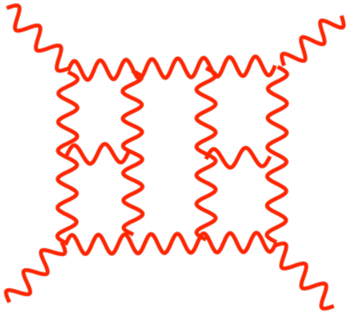
Based on this a reasonable person would conclude that $N = 8$ supergravity almost certainly diverges at 7 loops in $D = 4$.

Same methods also predict:

- **$N = 8$ sugra should diverge at 7 loops in $D = 4$**
- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$**
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$**
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$**

$\mathcal{N} = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Johannson, Roiban



~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same.

Place your bets:

- **At 5 loops in $D = 24/5$ does $\mathcal{N} = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $\mathcal{N} = 8$ supergravity diverge?**

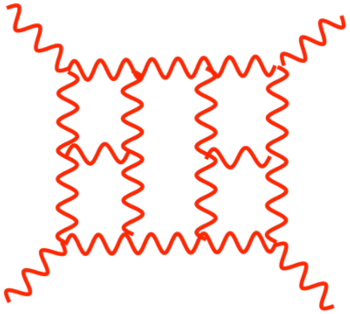
5 loops

Kelly Stelle:
English wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$\mathcal{N} = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Johannson, Roiban



~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same

Place your bets:

- **At 5 loops in $D = 24/5$ does $\mathcal{N} = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $\mathcal{N} = 8$ supergravity diverge?**

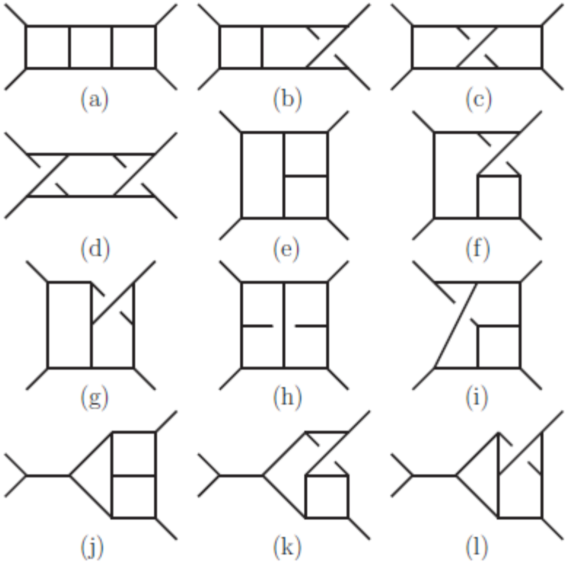
7 loops

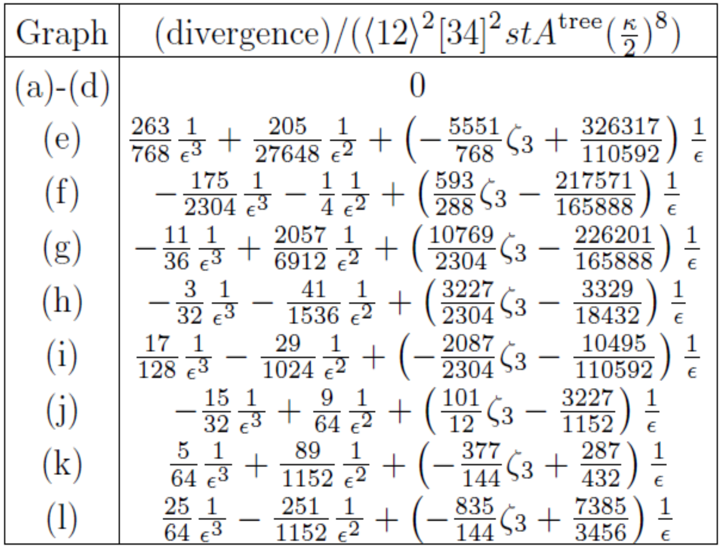
David Gross:
California wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$\mathcal{N} = 4$ Supergravity UV Cancellation

ZB, Davies, Dennen, Huang





Graph	(divergence)$/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

All **three-loop divergences and subdivergences cancel completely!**

Still no symmetry explanation, despite valiant attempt.

Bossard, Howe, Stelle; ZB, Davies, Dennen

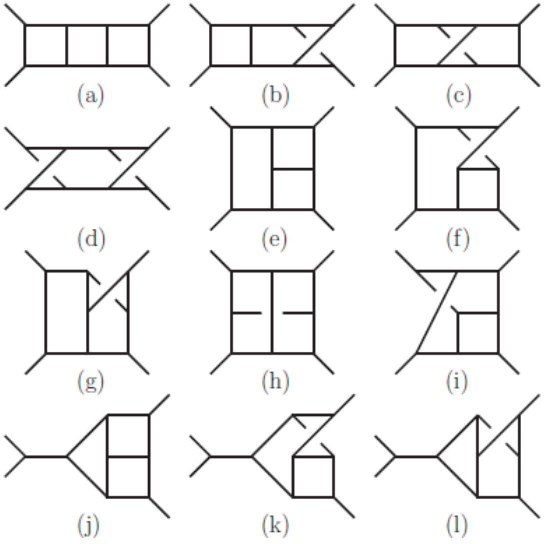
A pity we did not bet on this theory!
Prediction based supergravity imply divergences

Enhanced UV Cancellations

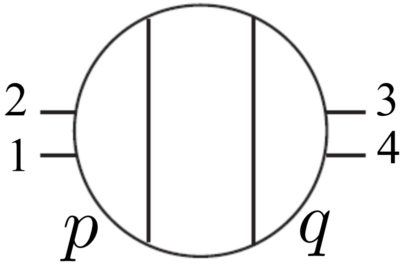
ZB, Davies, Dennen (2014)

Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.

- **By definition this is an enhanced cancellation.**
- **Not the way gauge theory works.**



***N* = 4 sugra**



already log divergent

***N* = 4 sugra: pure YM x *N* = 4 sYM**

$$n_i \sim s^3 t A_4^{\text{tree}} (p \cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

3 loop UV finiteness of *N* = 4 supergravity proves existence of "enhanced cancellation" in supergravity theories.

N = 5 Supergravity at Four Loops

ZB, Davies and Dennen

We also calculated four-loop divergence in *N* = 5 supergravity.

Industrial strength software needed: FIRE5 and C++

Crucial help from FIRE5 and (Smirnov)^2

***N* = 5 sugra: (*N* = 4 sYM) x (*N* = 1 sYM)**

***N* = 4 sYM**

***N* = 1 sYM**

Diagrams necessarily UV divergent.

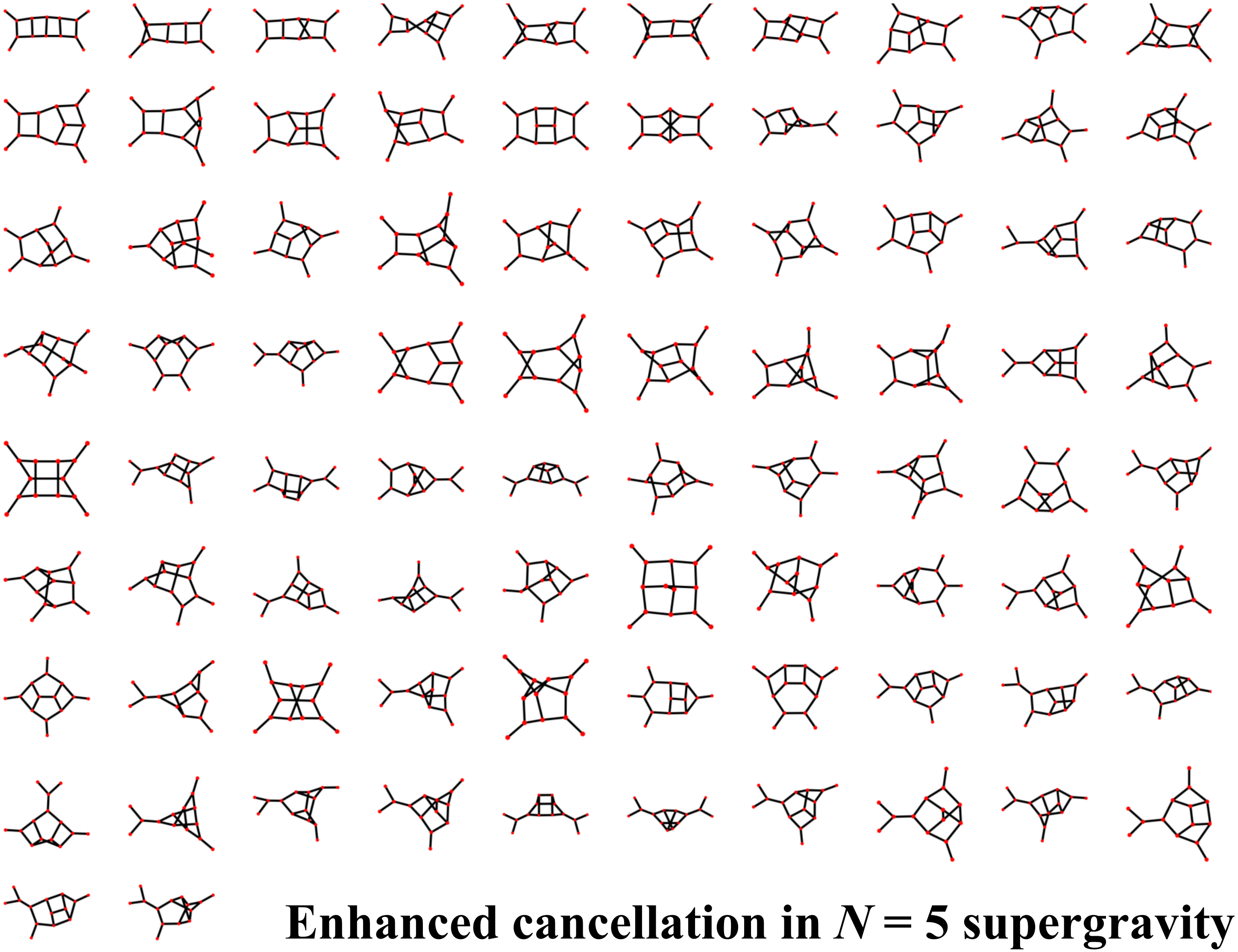
***N* = 5 supergravity has no divergence at four loops.**

Another example of an “enhanced cancellations”.

A pity we did not bet on this theory as well!

82 nonvanishing numerators in BCJ representation

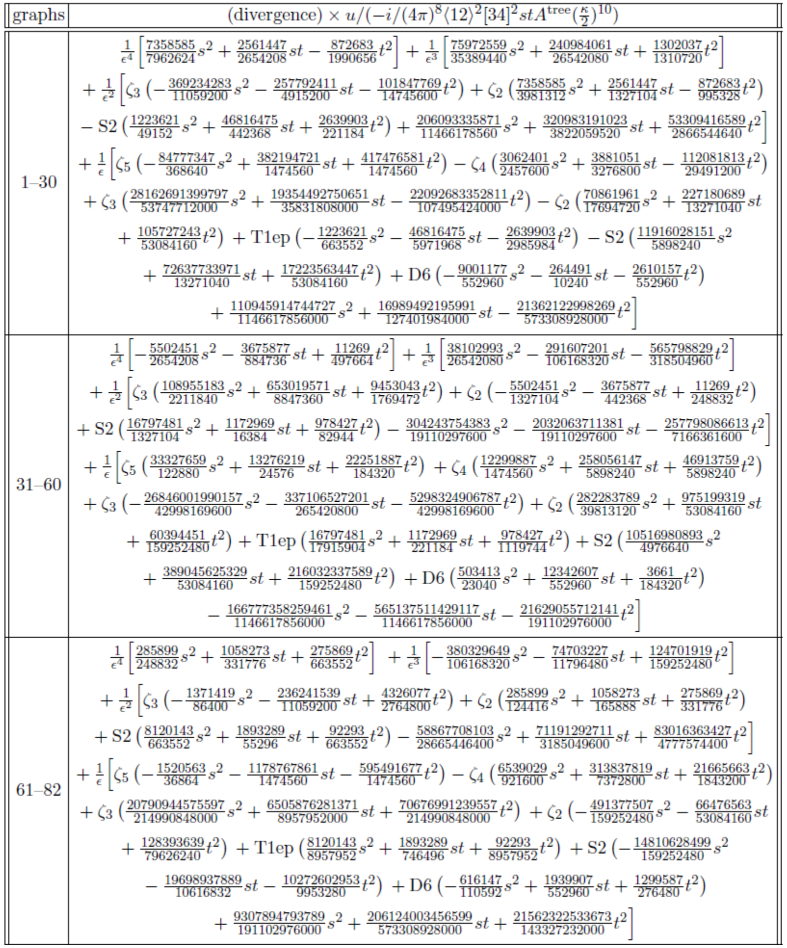
ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)



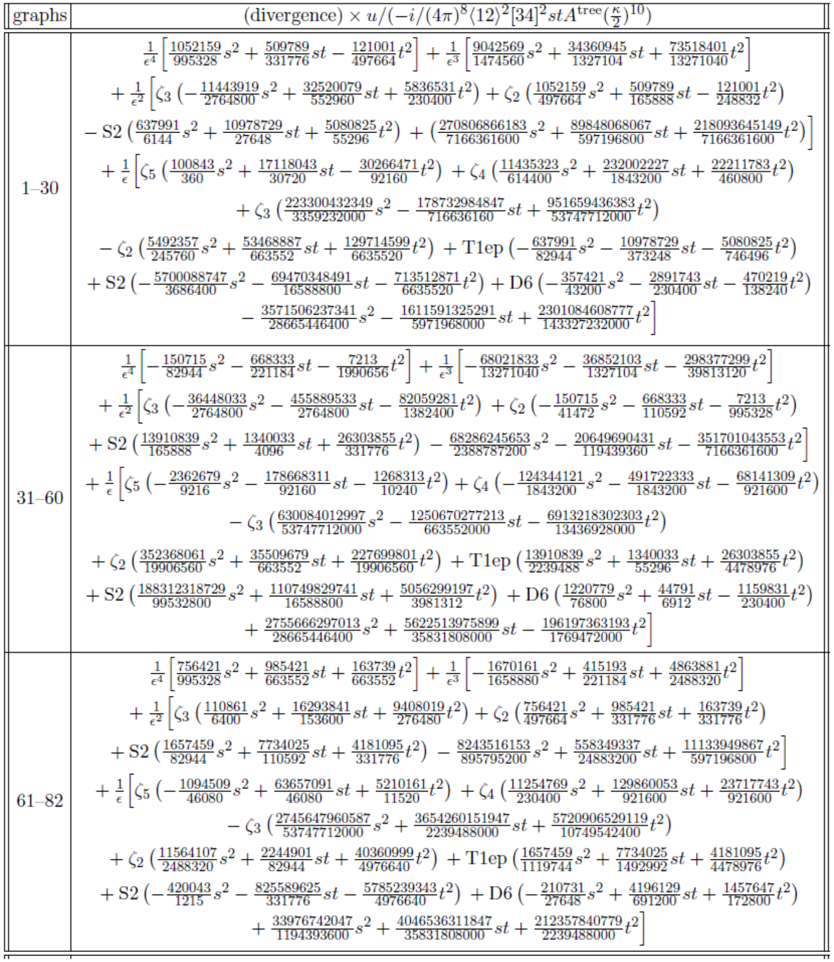
Enhanced cancellation in $\mathcal{N} = 5$ supergravity

$\mathcal{N} = 5$ supergravity at Four Loops

ZB, Davies and Dennen (2014)



graphs	(divergence) $\times u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2]+\frac{1}{\epsilon^3}[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2)+\zeta_2(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2)$ $-\mathrm{S2}(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2)-\zeta_4(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2)$ $+\zeta_3(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2)-\zeta_2(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st$ $+\frac{105727243}{53084160}t^2)+\mathrm{T1ep}(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2)-\mathrm{S2}(\frac{11916028151}{5898240}s^2$ $+\frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2)+\mathrm{D6}(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2)$ $+\frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2]$
31–60	$\frac{1}{\epsilon^4}[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2)+\zeta_2(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2)$ $+\mathrm{S2}(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2)+\zeta_4(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2)$ $+\zeta_3(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2)+\zeta_2(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st$ $+\frac{60394451}{159252480}t^2)+\mathrm{T1ep}(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2)+\mathrm{S2}(\frac{10516980893}{4976640}s^2$ $+\frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2)+\mathrm{D6}(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2)$ $-\frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2]$
61–82	$\frac{1}{\epsilon^4}[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2)+\zeta_2(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2)$ $+\mathrm{S2}(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2)-\zeta_4(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2)$ $+\zeta_3(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2)+\zeta_2(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st$ $+\frac{128393639}{79626240}t^2)+\mathrm{T1ep}(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2)+\mathrm{S2}(-\frac{14810628499}{159252480}s^2$ $-\frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2)+\mathrm{D6}(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2)$ $+\frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2]$



graphs	(divergence) $\times u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2)+\zeta_2(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2)$ $-\mathrm{S2}(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2)+(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2)]$ $+\frac{1}{\epsilon}[\zeta_5(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2)+\zeta_4(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2)$ $+\zeta_3(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2)$ $-\zeta_2(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2)+\mathrm{T1ep}(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2)$ $+\mathrm{S2}(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2)+\mathrm{D6}(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2)$ $-\frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2]$
31–60	$\frac{1}{\epsilon^4}[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2]+\frac{1}{\epsilon^3}[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2)+\zeta_2(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2)$ $+\mathrm{S2}(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2)+\zeta_4(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2)$ $-\zeta_3(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2)$ $+\zeta_2(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2)+\mathrm{T1ep}(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2)$ $+\mathrm{S2}(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2)+\mathrm{D6}(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2)$ $+\frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2]$
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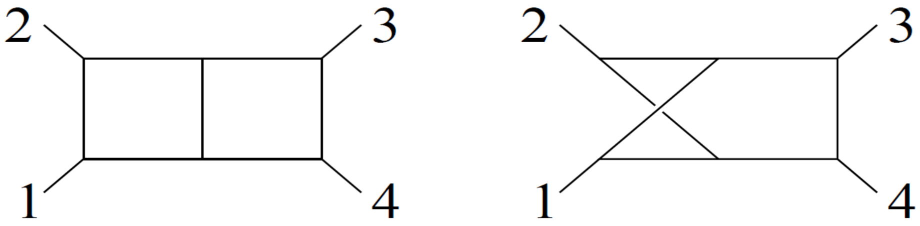
Adds up to zero: no divergence. Enhanced cancellations!
No standard symmetry explanation exists.

Where does new magic come from?

ZB, Davies, Dennen, Huang; Bossard, Howe, Stelle

To analyze we need a simpler example: Half-maximal supergravity in $D = 5$ at 2 loop.

Similar to $N = 4$, $D = 4$ sugra at 3 loops, except much simpler.



Quick summary:

- **— Finiteness in $D = 5$ tied to double-copy structure.**
- **— Cancellations in certain forbidden gauge-theory color structures imply hidden UV cancellations in supergravity, even when no standard symmetry explanation.**

Double copy structure implies extra cancellations!

Unfortunately, our 1, 2 loop proof not easy to extend beyond 2 loops.

The 4 loop Divergence of $N = 4$ Supergravity

ZB, Davies, Dennen, A.V. Smirnov, V.A. Smirnov

4 loops similar to 3 loops except we need industrial strength software: FIRE5 + special purpose C++ code.

dim. reg. UV pole

$$\boxed{\mathcal{M}^{\text{4-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}}$$

kinematic factor

$D = 4 - 2\epsilon$

It diverges but it has strange properties:

- **Contributions to helicity configurations that vanish were it not for a quantum anomaly in $U(1)$ subgroup of duality symmetry.**
- **These helicity configuration have vanishing integrands in $D = 4$. Divergence is 0/0. Anomaly-like behavior not found in $N \geq 5$ sugra.**

Carrasco, Kallosh, Tseytlin and Roiban

Motivates closer examination of divergences.
Want simpler example: Pure Einstein gravity is simpler.

Pure Einstein Gravity

Standard argument for 1 loop finiteness of pure gravity:

't Hooft and Veltman (1974)

$\cancel{R^2}$ $\cancel{R_{\mu\nu}^2}$

Divergences vanish by equation of motion and can be eliminated by field redefinition.

$\cancel{R_{\mu\nu\rho\sigma}^2}$

In *D* = 4 topologically trivial space, Gauss-Bonnet theorem eliminates Riemann square term.

$$\int d^4x\, \sqrt{-g}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) = 32\pi^2\chi$$

Euler Characteristic.

Pure gravity divergence with nontrivial topology:

Capper and Duff; Tsao ; Critchley; Gibbons, Hawking, Perry Goroff and Sagnotti, etc

$$\mathcal{L}^{\rm GB} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R_{\mu\nu} + R_{\mu\nu\rho\sigma}^2)$$

graviton **scalar** **antisym. tensor** **3 form tensor** **Gauss-Bonnet**

Related to "trace anomaly". (Also called conformal or Weyl anomaly.)

Gauss-Bonnet one-loop divergence is "evanescent"

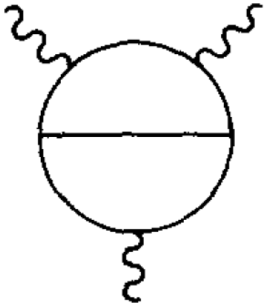
Two-Loop Pure Gravity

By two loops there is a valid *R*^{3} counterterm and corresponding divergence.

Goroff and Sagnotti (1986); Van de Ven (1992)

Divergence in pure Einstein gravity (no matter):

$$D = 4 - 2\epsilon$$



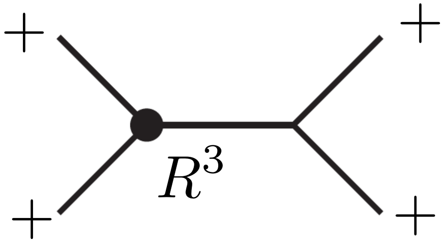
UV divergence

$$\mathcal{L}^{R^3} = \frac{209}{2880}\frac{1}{(4\pi)^4}\frac{1}{2\epsilon} R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}$$

On surface nothing weird going on.

However, when we apply modern tools we find results are subtle and weird, just like in *N* = 4 supergravity, once you probe carefully.

Two Loop Identical Helicity Amplitude



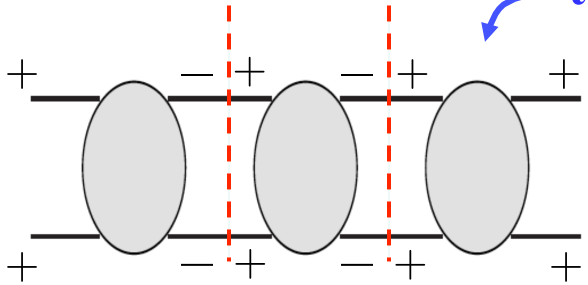
Pure gravity identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

$$D = 4 - 2\epsilon$$

$$\mathcal{M}^{R^3}\Big|_{\text{div.}} = \frac{209}{24\epsilon}\,\mathcal{K}$$

$$\mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

Curious feature:



tree amplitude vanishes

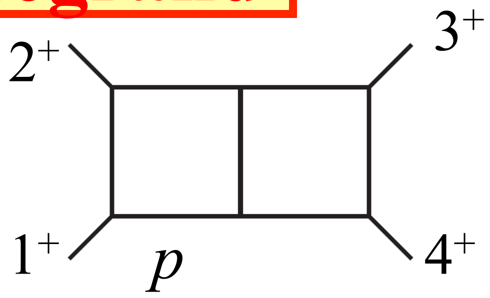
- Integrand vanishes for four-dimensional loop momenta.
- Nonvanishing because of ϵ-dimensional loop momenta.

Bardeen and Cangemi pointed out nonvanishing of identical helicity is connected to an anomaly in self-dual sector.

A surprise:

Divergence is *not* generic but appears tied to anomaly-like behavior.

Full Two-Loop Integrand

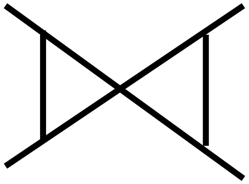


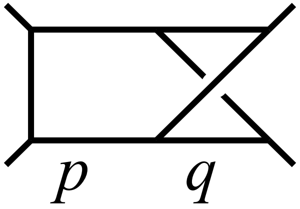
Using spinor helicity very compact:

$$\begin{aligned} n = {} & \frac{D_s(D_s-3)}{2}(\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2)^2 - \frac{D_s(D_s-6)}{2}\lambda_p^2\lambda_q^2\lambda_{p+q}^2(\lambda_p^2 + \lambda_q^2 + \lambda_{p+q}^2) \\ & + 12D_s((\lambda_p\cdot\lambda_q)^2 - \lambda_p^2\lambda_q^2)(\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2) + 144((\lambda_p\cdot\lambda_q)^2 - \lambda_p^2\lambda_q^2)^2\,, \end{aligned}$$

Bow-tie and nonplanar contributions similar:

$$p_i = p_i^{(4)} + \lambda_i$$

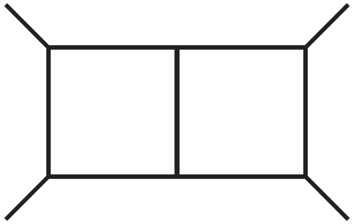




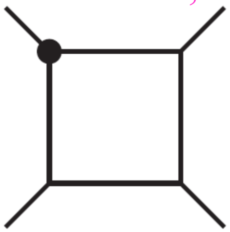
- **Integrand vanishes for *D* = 4 loop momenta:** λ^8
- **Upon integration ultraviolet divergent.**
- **Awesome simplicity in a seemingly impossibly complicated theory.**

Pure Gravity Divergence

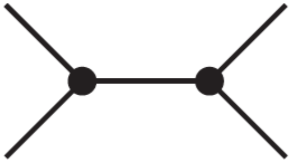
ZB, Cheung, Chi, Davies, Dixon and Nohle



2 loop bare



single GB subdivergence



double GB subdivergence

$D = 4 - 2\epsilon$

$$\left.\begin{aligned} \mathcal{M}_4^{\text{2-loop}}\Big|_{\text{div.}} &= -\frac{1}{\epsilon}\frac{3431}{5400}\mathcal{K} \\ \mathcal{M}_4^{\text{1-loop GB}}\Big|_{\text{div.}} &= \frac{1}{\epsilon}\frac{689}{675}\mathcal{K} \\ \mathcal{M}_4^{\text{tree GB}^2}\Big|_{\text{div.}} &= \frac{1}{\epsilon}\frac{5618}{675}\mathcal{K} \end{aligned}\right\} \qquad \mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\boxed{\frac{209}{24}}\mathcal{K}$$

Goroff and Sagnotti divergence reproduced

Surprise: Evanescent Gauss-Bonnet (GB) operator crucial part of UV structure. Dependence on trace anomaly!

Meaning of Divergence?

What does the divergence mean?

$$\Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$$

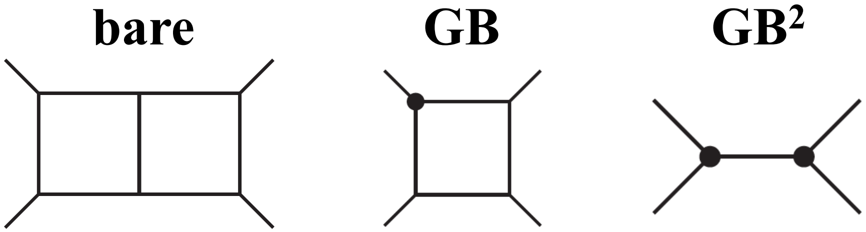
Adding n_3 3-form field offers good way to understand this:

- **On the one hand, no degrees of freedom in $D = 4$, so no change in divergence expected.**
- **On the other hand, the trace anomaly is affected, so expect change in divergence.**

- **Note that 3 form proposed as way to dynamically neutralize cosmological constant.**

Brown and Teitelboim; Bousso and Polchinski

	$1/\epsilon$
bare	$-\frac{3431}{5400} - \frac{199 n_3}{30} + 6 n_3^2$
GB	$\frac{4\cdot 53-180n_3}{360}\cdot\frac{2\cdot(13+180n_3)}{15}$
GB^2	$24\left(\frac{4\cdot 53-180n_3}{360}\right)^2$
total	$\frac{209}{24} - \frac{15}{2} n_3$



Divergence depends on nondynamical 3-form fields!

But wait: what about finite parts?

Scattering Amplitudes

Pure Gravity:

IR singularities subtracted and independent of 3 form

$$\mathcal{M}_G^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big[\boxed{\frac{1}{\epsilon}\frac{209}{24}stu} + \frac{117617}{21600}stu + \left(\frac{1}{10}stu - \frac{1}{60}s^3\right)\log\left(\frac{-s}{\mu^2}\right) + \frac{1}{120}\left(s^2+t^2+u^2\right)s\log^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\Big]$$

Gravity + 3 Form:

$$\mathcal{M}_{G3}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big[\boxed{\frac{1}{\epsilon}\frac{29}{24}stu} + \frac{411617}{21600}stu + \left(\frac{1}{10}stu - \frac{1}{60}s^3\right)\log\left(\frac{-s}{\mu^2}\right) + \frac{1}{120}\left(s^2+t^2+u^2\right)s\log^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\Big]$$

divergences different. logarithms identical!

- **Value of divergence not physical. Absorb into counterterm.**
- **3 form is a Cheshire Cat field: scattering unaffected.**

Similar results comparing scalar and two-forms.

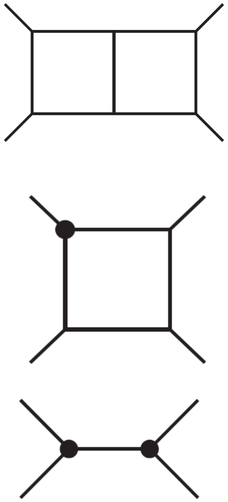
Results consistent with quantum equivalence under duality.
Firmly in quantum equivalence camp.

Duff and van Nieuwenhuizen; Siegel; Fradkin and Tseytlin; Grisaru, Nielsen, Siegel, Zanon, etc

$N = 1$ Supergravity

ZB, Chi, Dixon, Edison (to appear)

Divergence violates susy ward identity even though regulator should be supersymmetric! Due to trace anomaly.



Result for $N = 1$ supergravity with 1 matter multiplet

$$\mathcal{M}_4^{\mathrm{total}}\Big|_{\mathrm{div.}} = \frac{1}{\epsilon}\,\frac{81871}{21600}\,\mathcal{K} + 0\ln(\mu^2)\mathcal{K} + \mathrm{finite}$$

Very strange, but no stranger than earlier results.

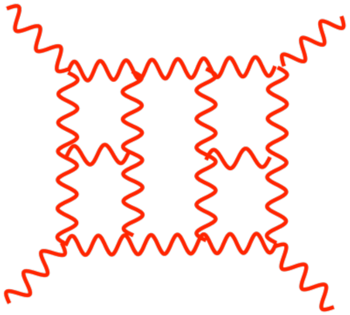
Have no fear: no physical effect! Local counterterm eats the divergence restoring susy.

Still working on case with no matter multiple, but no reason to expect different outcome.

New Directions in Gravity Loops

If you want to solve a difficult problem get an army of energetic young people to help with new ideas:

- **Better understanding and applications of BCJ duality.**
  Chiodaroli, Gunaydin, Johansson and Roiban,; Johannsson, Ochirov; O'Connell, Montiero, White; ZB, Davies, Nohle; Boels, Isermann, Monteiro, and O'Connel; Mogull and O'Connell, He, Monteiro, and Schlotterer
- **Scattering equations and double-copy relations.**
  Cachazo, He, Yuan
- **Twistor strings now at loop level for *N* = 8 supergravity.**
  Adamo, Casali and Skinner; Geyer, Mason, Monteiro and Tourkine
- **New ideas on unitarity cuts based on Feynman Tree Theorem**
  Baadsgaard, Bjerrum-Bohr, Bourjaily, Caron-Huot, Damgaard and Feng
- **Important advances in related string theory amplitudes.**
  Carlos Mafra and Oliver Schlotterer
- **Nonplanar analytic hints from Amplituhedron.**
  ZB, Hermann, Litsey, Stankowicz, Trnka
- **Awesome equation solver. Millions of equations encountered at 5 loops can be dealt with! Very cool algorithm!**
  Schabinger and von Manteuffel



Outlook

- We have only scratched the surface. Multi-loop gravity very rich.
- “Reports of the death of supergravity are an exaggeration”

Stephan Hawking (with help from Mark Twain)

- UV finiteness of supergravity, given up for dead twice, is back in business, with new surprises: *Enhanced UV cancellations.*
- I don’t know if this will lead to a completely satisfactory description of nature via supergravity. At least people are looking again at this possibility and we uncovered some interesting things along the way.

Summary

- Modern amplitudes approach is a powerful tool for quantum gravity. Is it possible to have perturbatively UV finite versions of Einstein gravity?

- Remarkable connection between gauge and gravity theories:
  - color ⟷ kinematics.
  - gravity ~ (gauge theory)^2

- Pure supergravities surprisingly tame in the UV. New phenomenon: *Enhanced cancellations.*

- Strange anomaly-like behavior of divergences in gravity. Strange delinking of divergences from scaling behavior.

Supersymmetric versions of Einstein's General Relativity are surprisingly tame in the ultraviolet. Expect that the curious story will continue.

Extra Slides

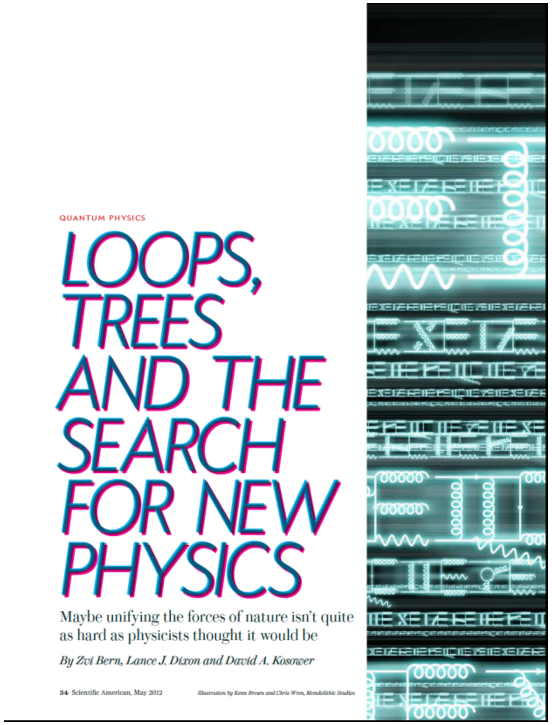
Further Reading

If you wish to read more see following non-technical descriptions.

Hermann Nicolai, *PRL Physics Viewpoint*, "Vanquishing Infinity"
http://physics.aps.org/articles/v2/70

Z. Bern, L. Dixon, D. Kosower,
May 2012 *Scientific American*,
"Loops, Trees and the Search for New Physics"

Anthony Zee, *Quantum Field Theory in a Nutshell*, 2nd Edition is first textbook to contain modern formulation of scattering and commentary on new developments. 4 new chapters.



QUANTUM PHYSICS

LOOPS, TREES AND THE SEARCH FOR NEW PHYSICS

Maybe unifying the forces of nature isn't quite as hard as physicists thought it would be

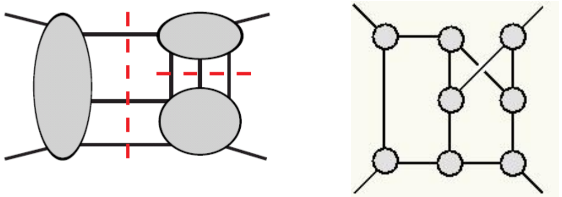
By Zvi Bern, Lance J. Dixon and David A. Kosower

34 Scientific American, May 2012

Illustration by Kenn Brown and Chris Wren, Mondolithic Studios

Our Basic Tools

We have powerful tools for computing scattering amplitudes in quantum gravity and for uncovering new structures:



- **Unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson , Kosower

- **Advanced loop integration technology.**

  Chetyrkin, Kataev and Tkachov; A.V. Smirnov; V. A. Smirnov, Vladimirov; Marcus, Sagnotti; Czakon; etc

- **Duality between color and kinematics.**

  ZB, Carrasco and Johansson

Many other tools and advances discussed in other talks that I won't discuss here.