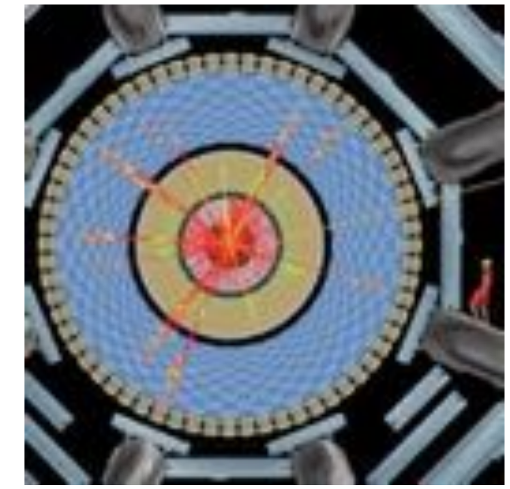
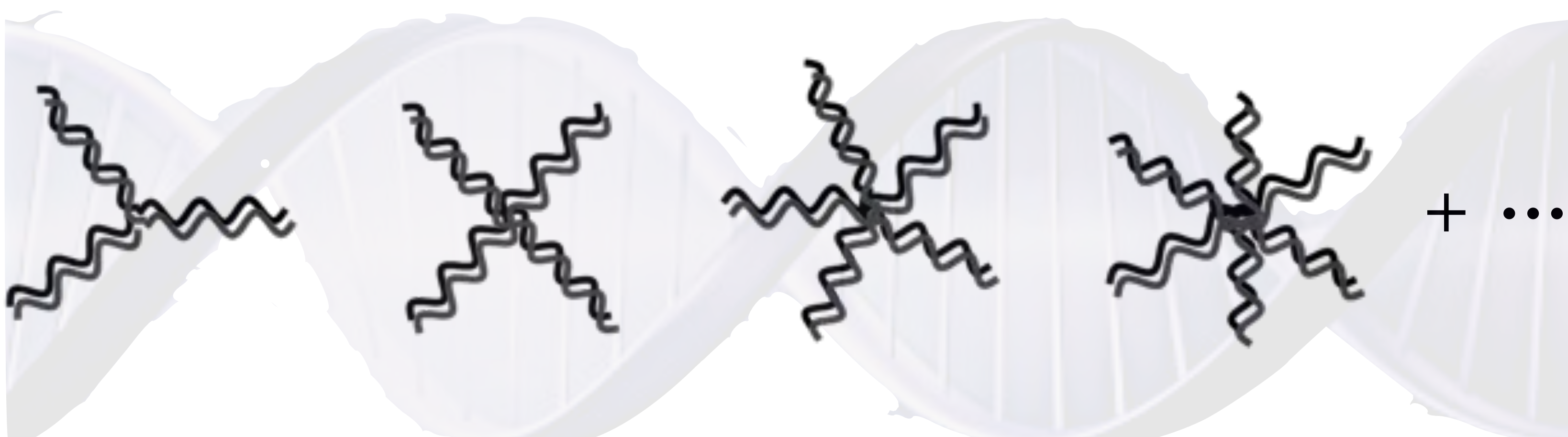
Double-copies from scattering

=



to rotating black holes;



=

a mostly positive take, with only two negatives

30 years of MHV

IPhT cea saclay

John Joseph M Carrasco

IAS

Fermilab

18 Mar 2016

MHV@ 30:

(some words of gratitude)

Not just an inspiration

Critical and USED

1. ***Testbed for ideas.***

3. ***4D satisfaction is necessary for D-dim claims.*** In combination with unitarity and (projections from) Nair superspace, MHV critical to how I probe all 4-D data from gauge theories I care about (c.f. http://inspirehep.net/record/816768)

Key Point: **MANY Theories are Double Copies**

Bi-Adjoint Scalar: color $\otimes$ color

Bern, de Freitas, Wong ('99), Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell

(S)YM (…(S)QCD…): color $\otimes$ spin-1

BCJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Steiberger; Feng et al; Mafra, Schlotterer, ('08-'11); Johansson, Ochirov

(S)Gr (…(S)Einstein-YM…): spin-1 $\otimes$ spin-1

KLT('86); BCJ ('08); Chiodaroli, Gunaydin, Johansson, Roiban; Johansson,Ochirov

NLSM: "color" $\otimes$ even-scalar

Chen, Du '13

(S)Born-Infeld: spin-1 $\otimes$ even-scalar

Cachazo, He, Yuan '14

Gallileon: even-scalar $\otimes$ even-scalar

Cachazo, He, Yuan '14

Open String: α' $\otimes$ spin-1

Broedel, Schlotterer, Steiberger

Closed String: spin-1 $\otimes$ α' $\otimes_{\alpha'}$ α' $\otimes$ spin-1

Broedel, Schlotterer, Steiberger

(see also talks of Henrik, Song, Ellis, and Louise)

Key Point: **MANY Theories are Double Copies**

Let me give you the two ``negatives'' first:

- I don't know why such a gauge choice exists

- I don't know how to exploit the benefits in all circumstances

Key Point: **MANY Theories are Double Copies**

Let me give you some positives:

+ Web of relationships between theories

+ Can exploit for technical simplicity in prediction
(see also Simon's and Henrik's talks)

+ Exposes a beautiful geometry in S-matrix

+ Each of the ``negatives'' is an opportunity
to learn about the language we use to
describe the universe.

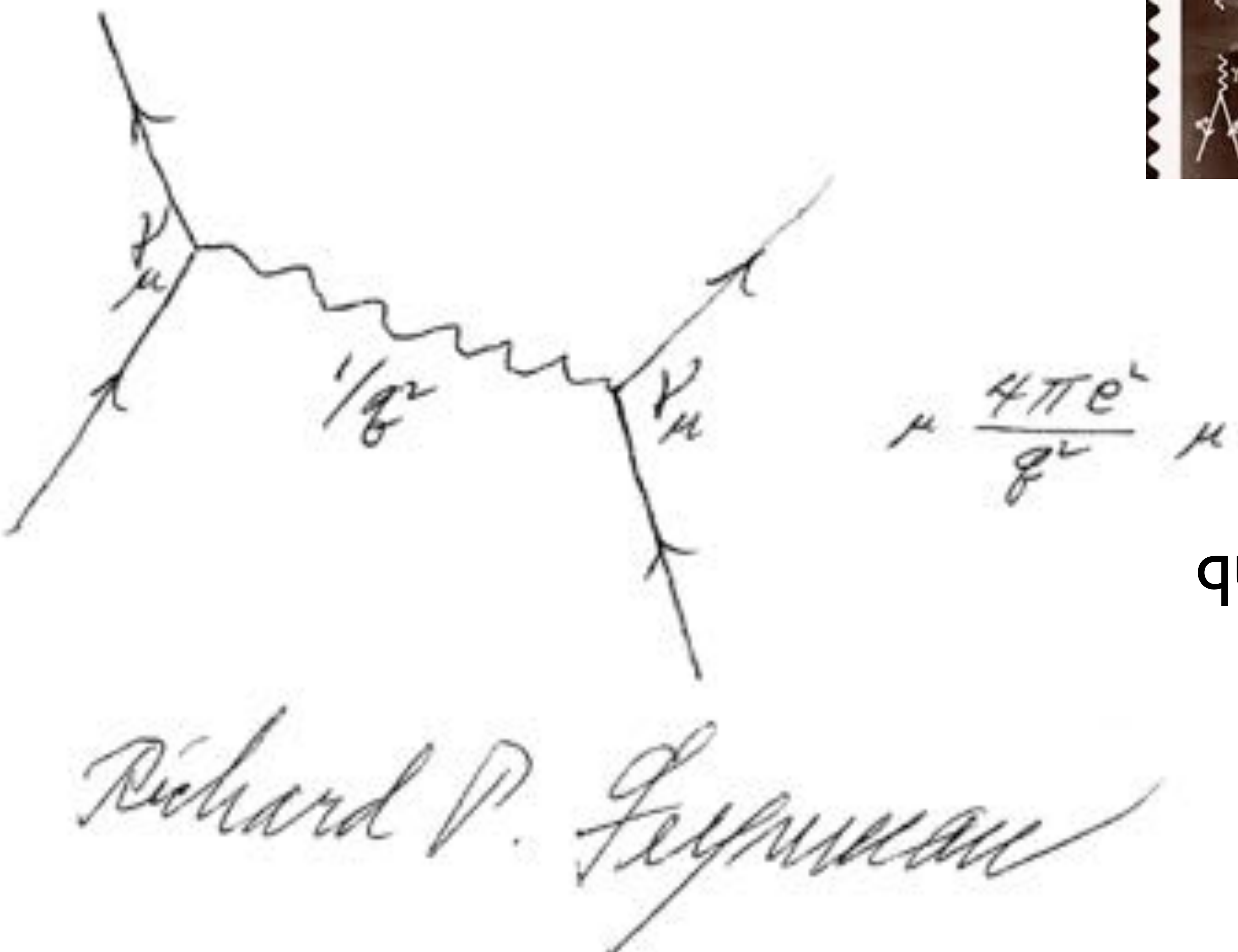
1. Need for Technical Simplicity

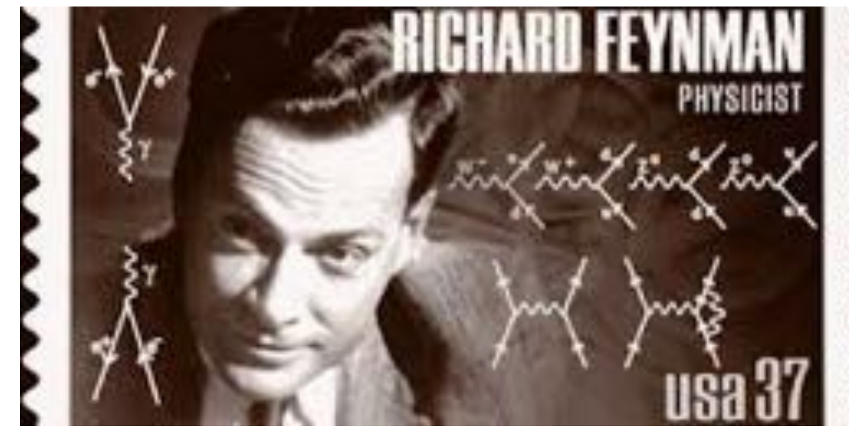
(the perils of infinite employment prospects)

(See also Zvi's talk!)

Complexity of Carrying Unphysical Information

"Do Feynman rules represent a useful solution??"





trees: semi-classical

loops: increasing quantum corrections

Off-shell three-graviton vertex:

$$
\begin{aligned}
&\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \to 2\eta^{\mu\tau}\eta^{\nu\sigma}k_1^{\lambda}k_1^{\rho} + 2\eta^{\mu\sigma}\eta^{\nu\tau}k_1^{\lambda}k_1^{\rho} - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1^{\lambda}k_1^{\rho} + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}k_1^{\sigma}k_1^{\rho} + 2\eta^{\lambda\sigma}\eta^{\mu\nu}k_1^{\tau}k_1^{\rho} + \eta^{\mu\tau}\eta^{\nu\sigma}k_2^{\lambda}k_1^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}k_2^{\lambda}k_1^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_2^{\mu}k_1^{\rho} + \\
&\eta^{\lambda\sigma}\eta^{\nu\tau}k_2^{\mu}k_1^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2^{\nu}k_1^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2^{\nu}k_1^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_3^{\mu}k_1^{\rho} + \eta^{\lambda\sigma}\eta^{\nu\tau}k_3^{\mu}k_1^{\rho} - \\
&\eta^{\lambda\nu}\eta^{\sigma\tau}k_3^{\mu}k_1^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_3^{\nu}k_1^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_3^{\nu}k_1^{\rho} - \eta^{\lambda\mu}\eta^{\sigma\tau}k_3^{\nu}k_1^{\rho} + \eta^{\lambda\nu}\eta^{\mu\tau}k_3^{\sigma}k_1^{\rho} + \\
&\eta^{\lambda\mu}\eta^{\nu\tau}k_3^{\sigma}k_1^{\rho} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_3^{\tau}k_1^{\rho} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_3^{\tau}k_1^{\rho} + 2\eta^{\mu\nu}\eta^{\rho\tau}k_1^{\lambda}k_1^{\sigma} + 2\eta^{\mu\nu}\eta^{\rho\sigma}k_1^{\lambda}k_1^{\tau} - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}k_1^{\sigma}k_1^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_1^{\sigma}k_1^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_1^{\sigma}k_1^{\tau} + \eta^{\mu\tau}\eta^{\nu\rho}k_1^{\sigma}k_2^{\lambda} + \eta^{\mu\rho}\eta^{\nu\tau}k_1^{\sigma}k_2^{\lambda} + \\
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&\eta^{\lambda\rho}\eta^{\nu\tau}k_1^{\sigma}k_2^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_1^{\sigma}k_2^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1^{\tau}k_2^{\mu} - \eta^{\lambda\rho}\eta^{\nu\sigma}k_1^{\tau}k_2^{\mu} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_1^{\tau}k_2^{\mu} + \\
&2\eta^{\nu\rho}\eta^{\sigma\tau}k_2^{\lambda}k_2^{\mu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_1^{\lambda}k_2^{\nu} + \eta^{\mu\sigma}\eta^{\rho\tau}k_1^{\lambda}k_2^{\nu} + \eta^{\lambda\tau}\eta^{\mu\rho}k_1^{\sigma}k_2^{\nu} - \eta^{\lambda\rho}\eta^{\mu\tau}k_1^{\sigma}k_2^{\nu} + \\
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&\eta^{\nu\sigma}\eta^{\rho\tau}k_1^{\lambda}k_3^{\mu} - \eta^{\nu\rho}\eta^{\sigma\tau}k_1^{\lambda}k_3^{\mu} + \eta^{\lambda\tau}\eta^{\nu\rho}k_1^{\sigma}k_3^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_1^{\sigma}k_3^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1^{\tau}k_3^{\mu} + \\
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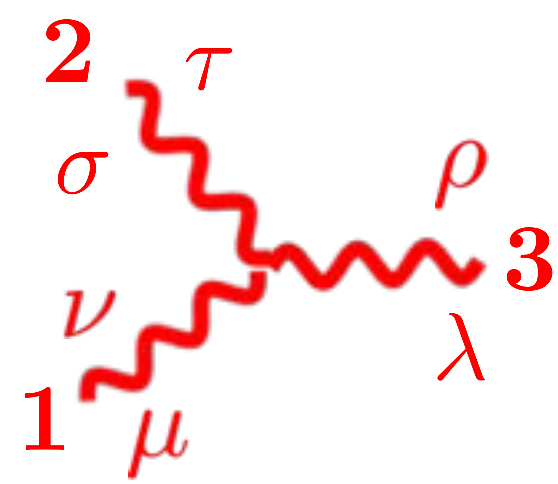
&\eta^{\lambda\nu}\eta^{\mu\rho}k_1^{\tau}k_3^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1^{\tau}k_3^{\sigma} + \eta^{\mu\tau}\eta^{\nu\rho}k_2^{\lambda}k_3^{\sigma} + \eta^{\mu\rho}\eta^{\nu\tau}k_2^{\lambda}k_3^{\sigma} - \eta^{\mu\nu}\eta^{\rho\tau}k_2^{\lambda}k_3^{\sigma} + \\
&\eta^{\lambda\tau}\eta^{\nu\rho}k_2^{\mu}k_3^{\sigma} + \eta^{\lambda\nu}\eta^{\rho\tau}k_2^{\mu}k_3^{\sigma} + \eta^{\lambda\tau}\eta^{\mu\rho}k_2^{\nu}k_3^{\sigma} + \eta^{\lambda\mu}\eta^{\rho\tau}k_2^{\nu}k_3^{\sigma} - \eta^{\lambda\tau}\eta^{\mu\nu}k_2^{\rho}k_3^{\sigma} + \\
&\eta^{\lambda\nu}\eta^{\mu\tau}k_2^{\rho}k_3^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\tau}k_2^{\rho}k_3^{\sigma} + 2\eta^{\lambda\rho}\eta^{\nu\tau}k_3^{\mu}k_3^{\sigma} + 2\eta^{\lambda\rho}\eta^{\mu\tau}k_3^{\nu}k_3^{\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}k_1^{\lambda}k_3^{\tau} + \\
&\eta^{\mu\rho}\eta^{\nu\sigma}k_1^{\lambda}k_3^{\tau} + \eta^{\lambda\nu}\eta^{\mu\rho}k_1^{\sigma}k_3^{\tau} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1^{\sigma}k_3^{\tau} + \eta^{\mu\sigma}\eta^{\nu\rho}k_2^{\lambda}k_3^{\tau} + \eta^{\mu\rho}\eta^{\nu\sigma}k_2^{\lambda}k_3^{\tau} - \\
&\eta^{\mu\nu}\eta^{\rho\sigma}k_2^{\lambda}k_3^{\tau} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_2^{\mu}k_3^{\tau} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_2^{\mu}k_3^{\tau} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_2^{\nu}k_3^{\tau} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_2^{\nu}k_3^{\tau} - \\
&\eta^{\lambda\sigma}\eta^{\mu\nu}k_2^{\rho}k_3^{\tau} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_2^{\rho}k_3^{\tau} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_2^{\rho}k_3^{\tau} + 2\eta^{\lambda\rho}\eta^{\nu\sigma}k_3^{\mu}k_3^{\tau} + 2\eta^{\lambda\rho}\eta^{\mu\sigma}k_3^{\nu}k_3^{\tau} - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}k_3^{\sigma}k_3^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_3^{\sigma}k_3^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_3^{\sigma}k_3^{\tau} - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot \\
&k_2 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_2 + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_2 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_2 - \\
&\eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_2 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_2 + \\
&2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_2 - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_3 + \\
&2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_3 - \\
&\eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_3 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - \\
&\eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_3 - \\
&\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2\cdot k_3 - \\
&\eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_2\cdot k_3 + \eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_2\cdot k_3 - \\
&\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2\cdot k_3 + \eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2\cdot k_3
\end{aligned}
$$

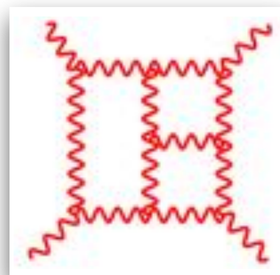
171 terms

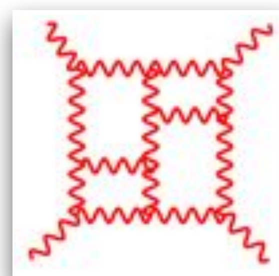


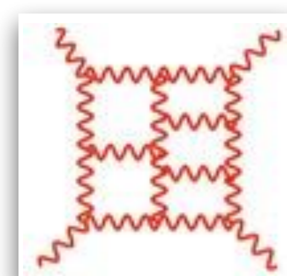
[DeWitt, 1967]

As Zvi told us textbook approach crumbles:

Feynman rules for a graviton: 171 terms per vertex
3 terms per edge

A single 3 loop diagram:  **~10^{20}** TERMS

4 loop diagram:  **~10^{26}** TERMS

5 loop diagram:  **~10^{31}** TERMS

BUT FINAL EXPRESSIONS ARE TRACTABLE

Vast majority of terms: unphysical freedom that must cancel

Some secrets obscured in the Lagrangian

Calculate with physical (on-shell) quantities: $k_i^2 = 0$

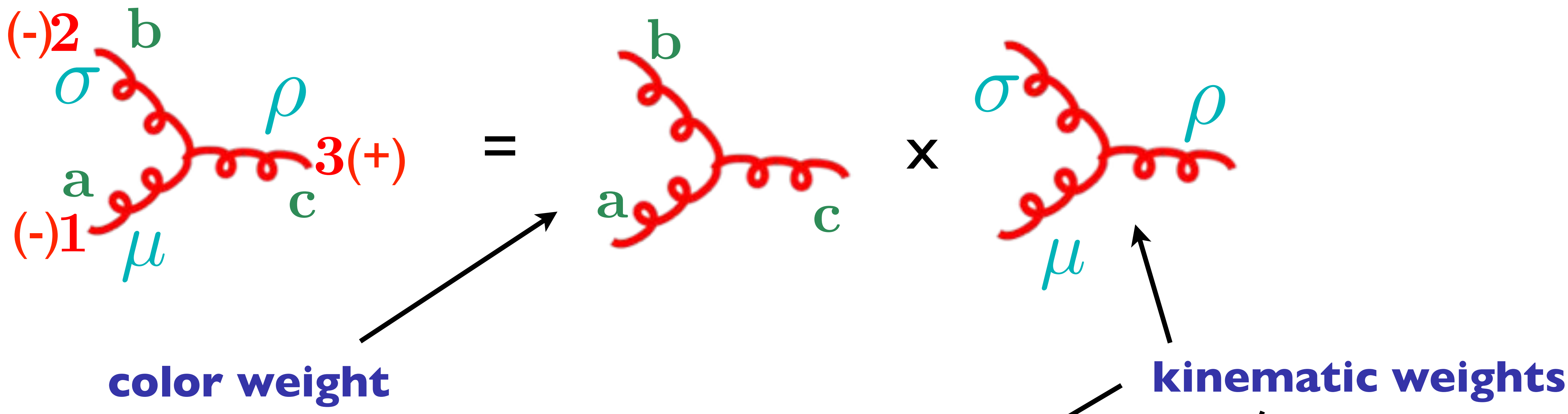
Physical (on-shell) **tree-level amplitudes contain all the information necessary to build** ***all*** **loop-level amplitudes** Bern, Dixon, Dunbar, and Kosower ('94,'95)

Bern, Dixon, and Kosower ('96)

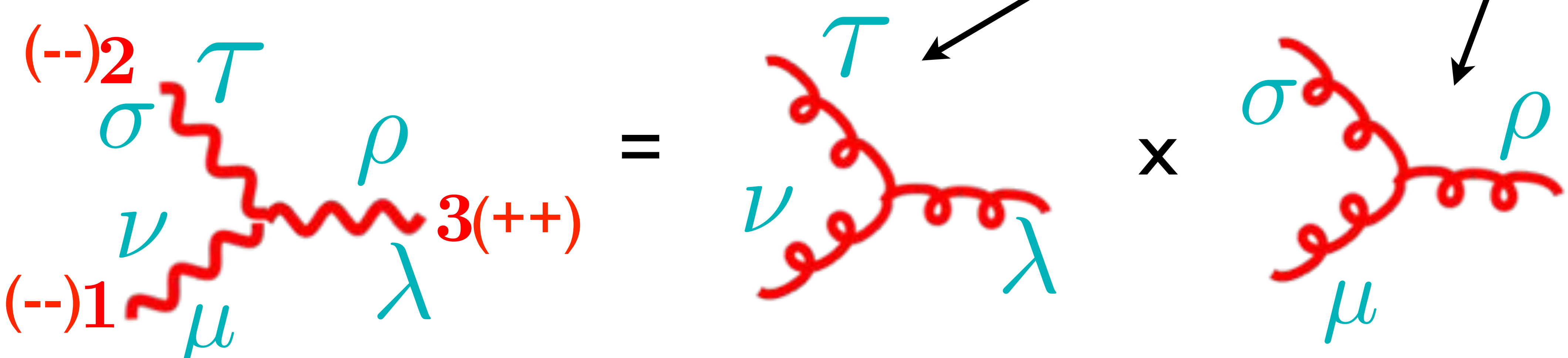
Physical (on-shell) **three-vertices contain all the information necessary to build** ***all*** **tree-level amplitudes** Britto, Cachazo, Feng, and Witten ('05)

$$k_i^2 = 0$$

Physical gluon 3-vertex: $\mathbf{f}^{\mathbf{abc}} \left(k_1{}^{\sigma} \eta^{\mu\rho} - k_2{}^{\mu} \eta^{\rho\sigma}\right)$



Physical graviton 3-vertex:

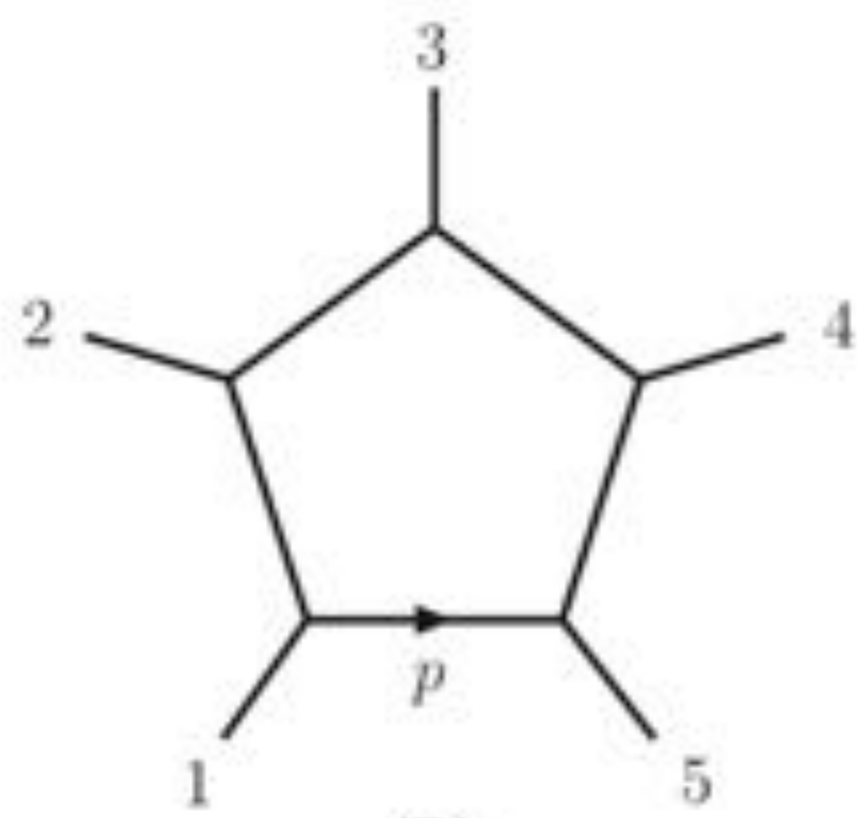


$$\left(\mathbf{k_1}{}^{\sigma} \eta^{\mu\rho} - \mathbf{k_2}{}^{\mu} \eta^{\rho\sigma}\right) \left(\mathbf{k_1}{}^{\tau} \eta^{\nu\lambda} - \mathbf{k_2}{}^{\nu} \eta^{\lambda\tau}\right)$$

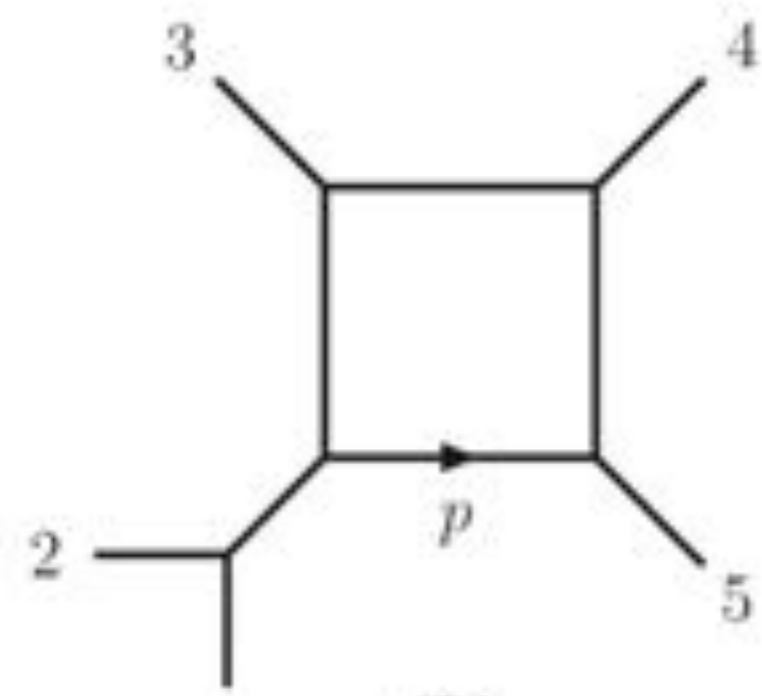
Complexity of Insisting on Local Representations

JJMC, Johansson (2011)

Five point 1-loop (no triangles, no bubbles)



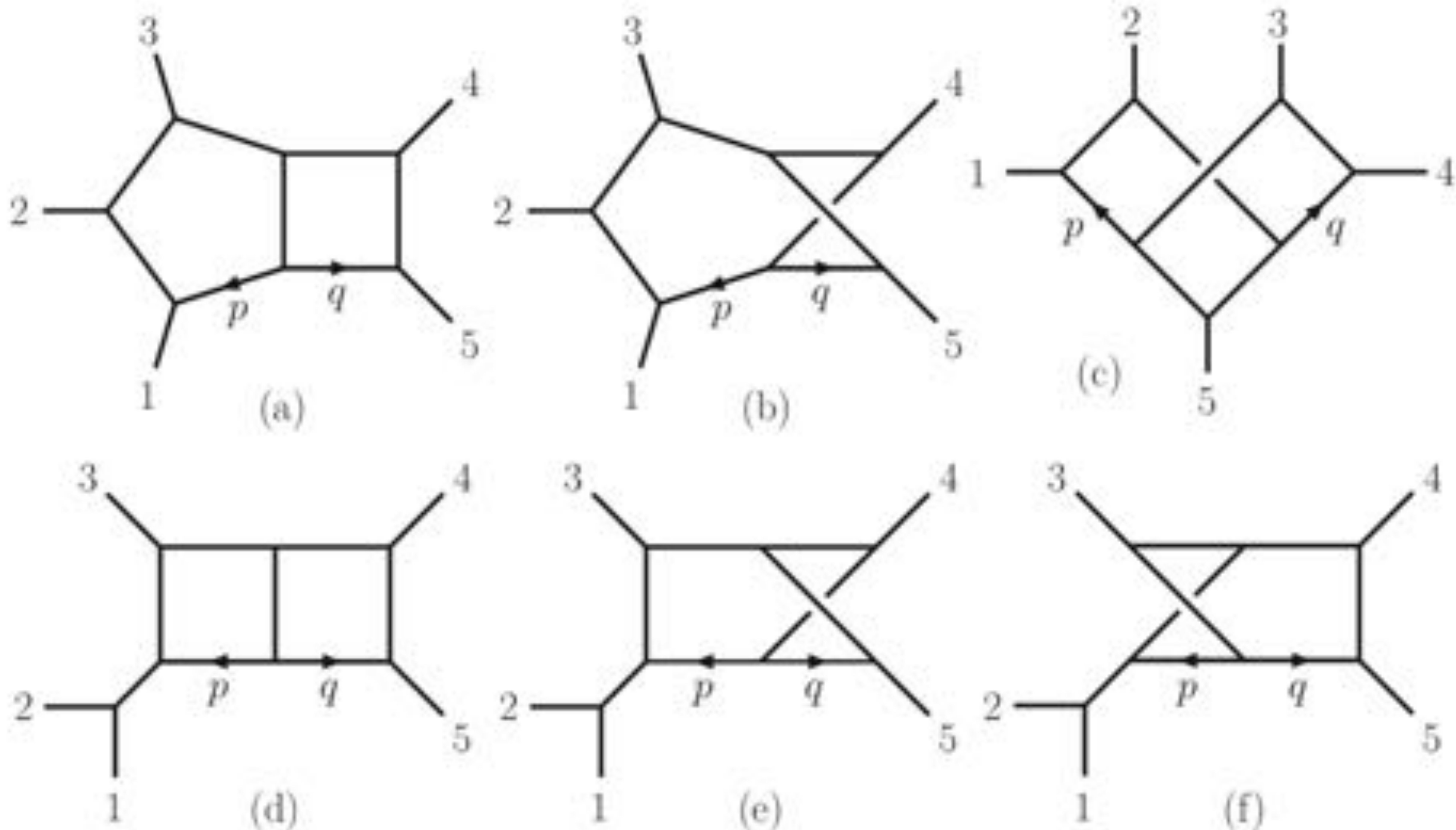
(P)



(B)

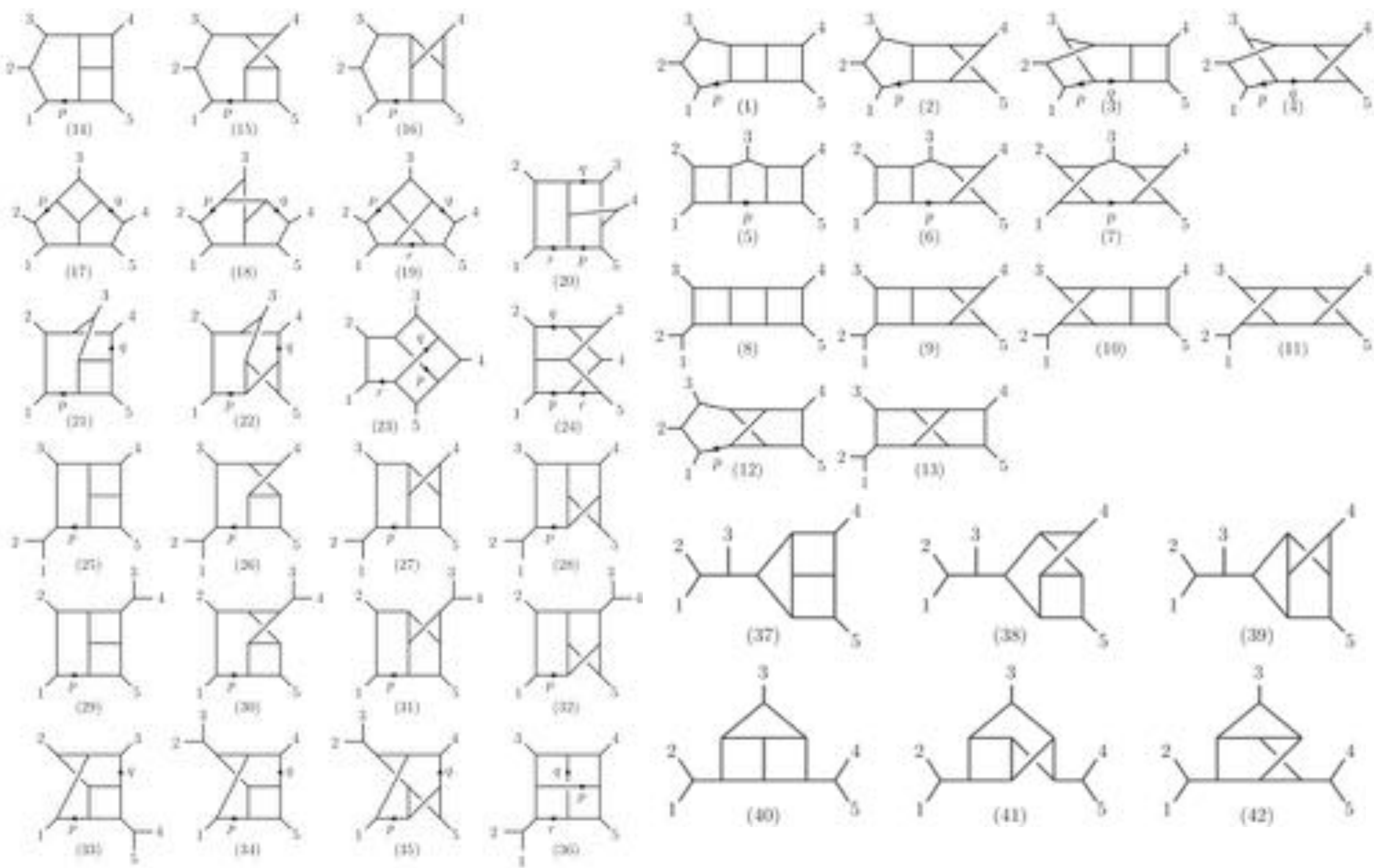
JJMC, Johansson (2011)

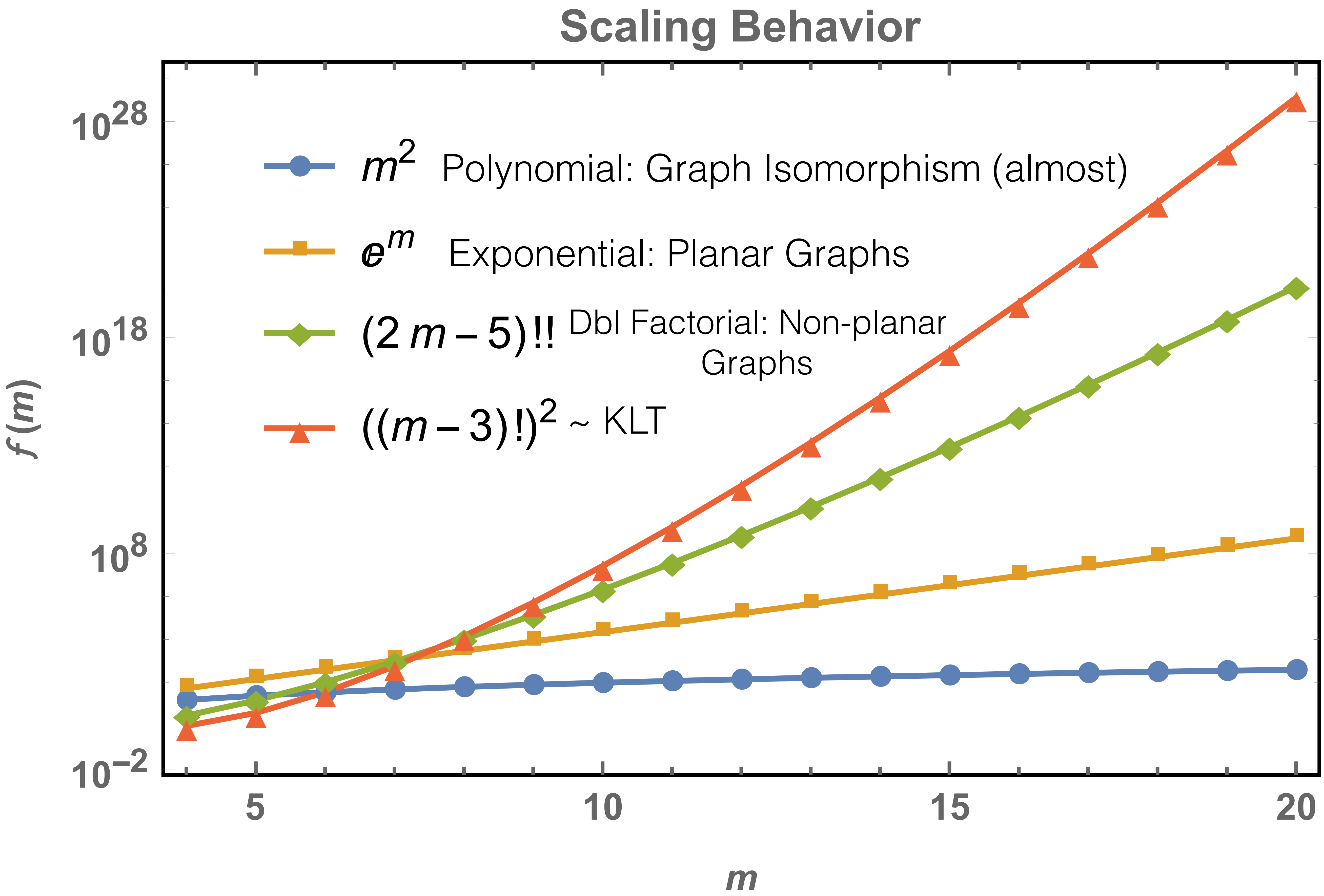
Five point 2-loop (no triangles, no bubbles)



Five point 3-loop (no bubbles, no triangles)

JJMC, Johansson (to appear)



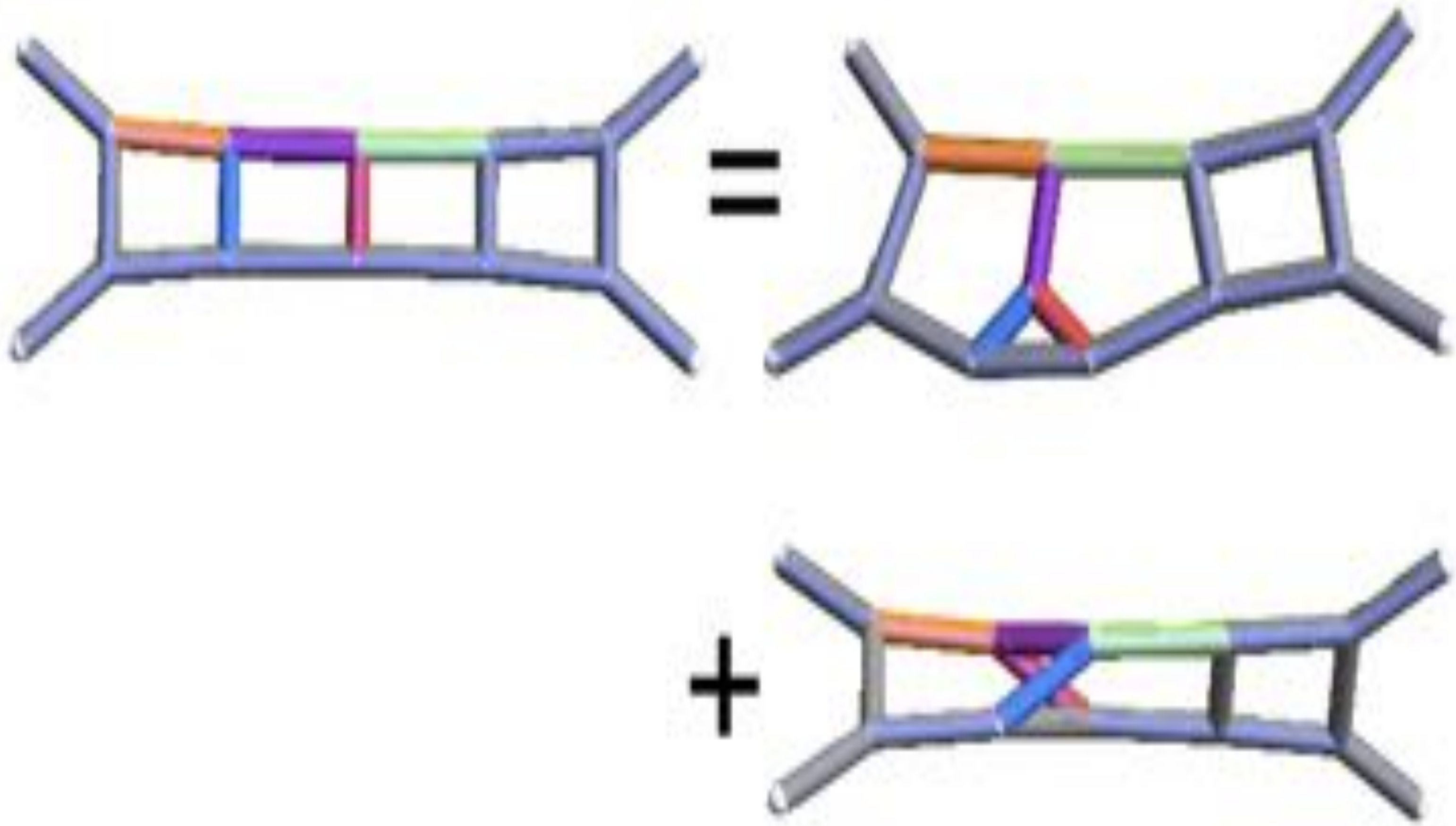


Scaling Behavior
10^{28}
10^{18}
10^{8}
10^{-2}
$f(m)$
5
10
15
20
m
m^2 Polynomial: Graph Isomorphism (almost)
e^m Exponential: Planar Graphs
$(2m-5)!!$ Dbl Factorial: Non-planar Graphs
$((m-3)!)^2$ ~ KLT

2. Can exploit Double Copy of YM

Bern, JJMC, Johansson ('08,'10)

Color and Kinematics dance together.



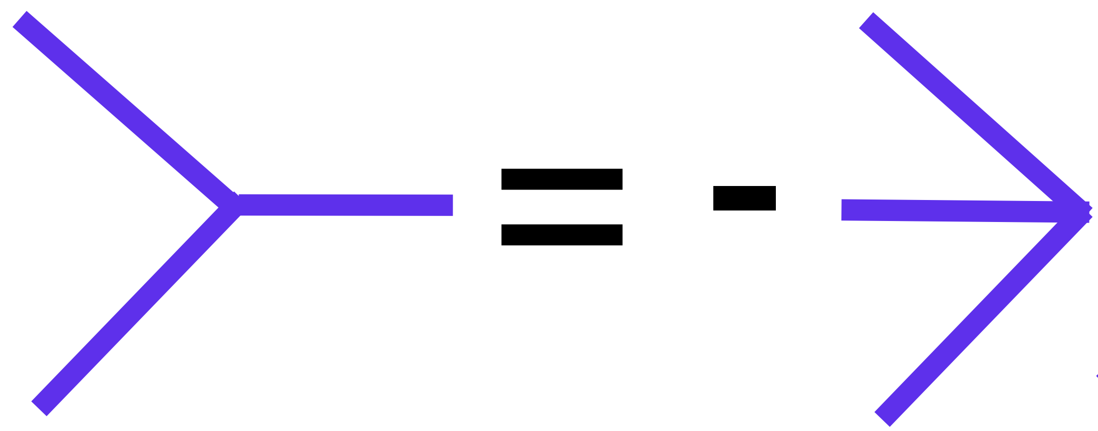
Solving Yang-Mills theories means solving Gravity theories.

Bern, JJMC, Johansson ('08,'10)

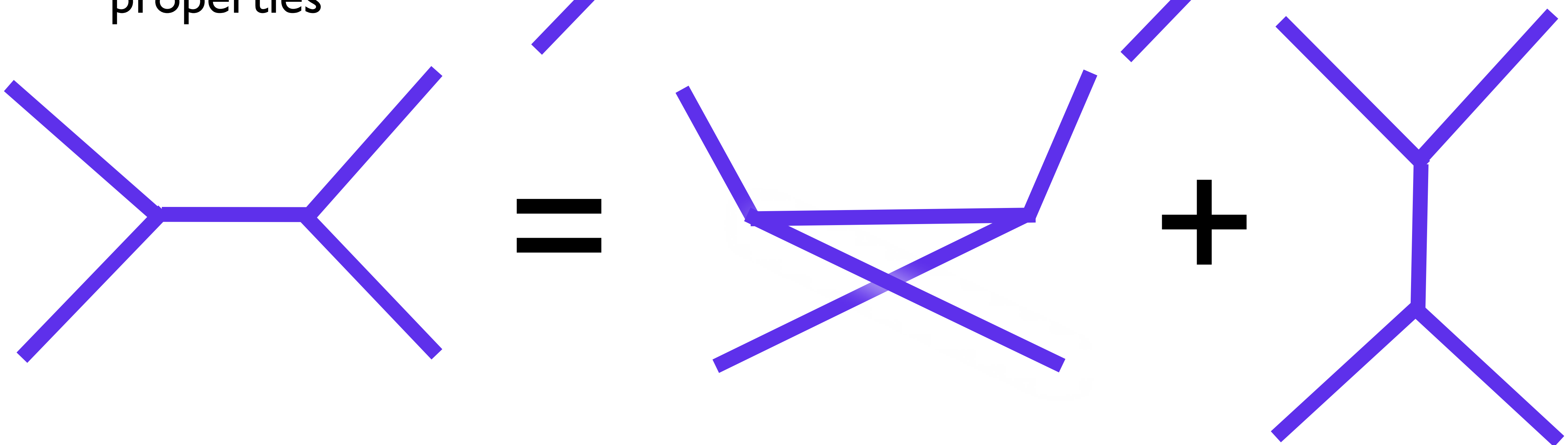
Generic D-dimensional YM theories have a fascinating structure at tree-level

$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G} \in \text{cubic}} \frac{c(\mathcal{G})\, n(\mathcal{G})}{D(\mathcal{G})}$$

Color factors and numerator factors satisfy similar lie algebra properties



Vertex Antisymmetry



Jacobi

Color-Kinematic Duality!

Bern, JJMC, Johansson ('08,'10)

Generic D-dimensional YM theories have a fascinating structure at tree-level

$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})n(\mathcal{G})}{D(\mathcal{G})}$$

YM: Color-Kinematic Duality, makes manifest gravitational double copy structure:

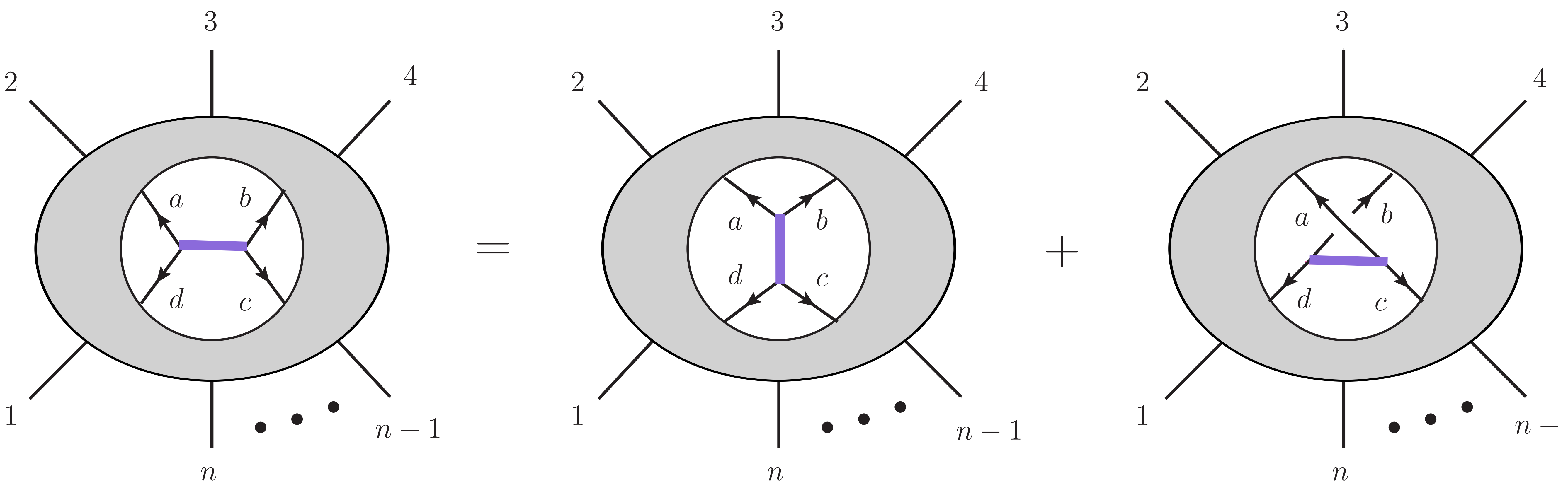
$$-iM_n^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$


GR = YM^2

Valid multi-loop generalization?

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$

CONJECTURE: for all graphs, can impose CK on every edge:



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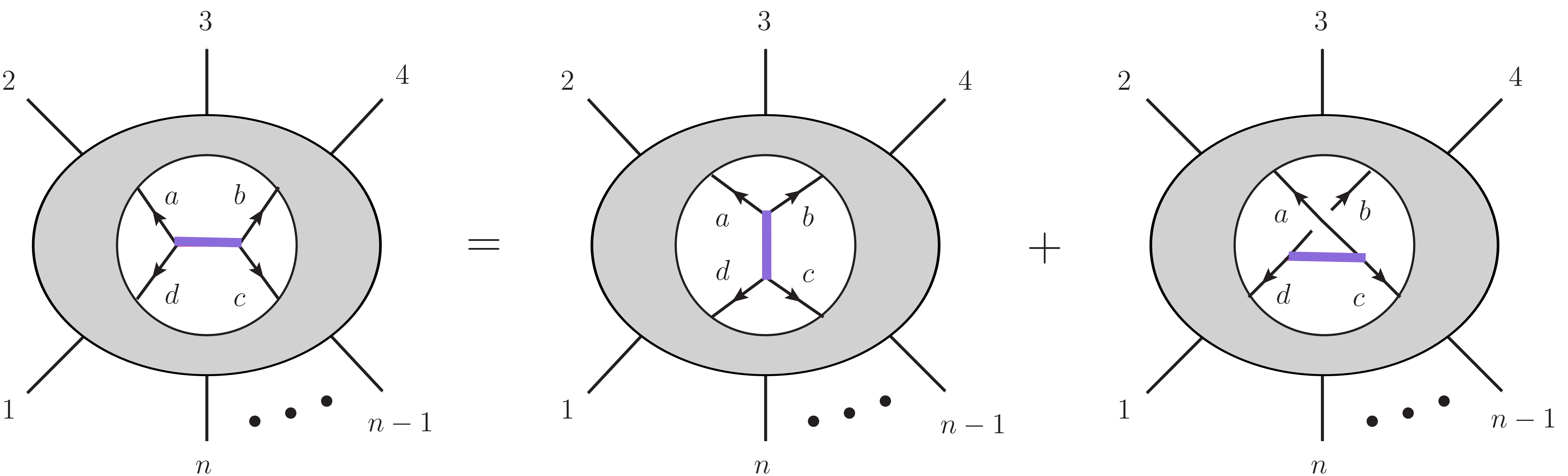
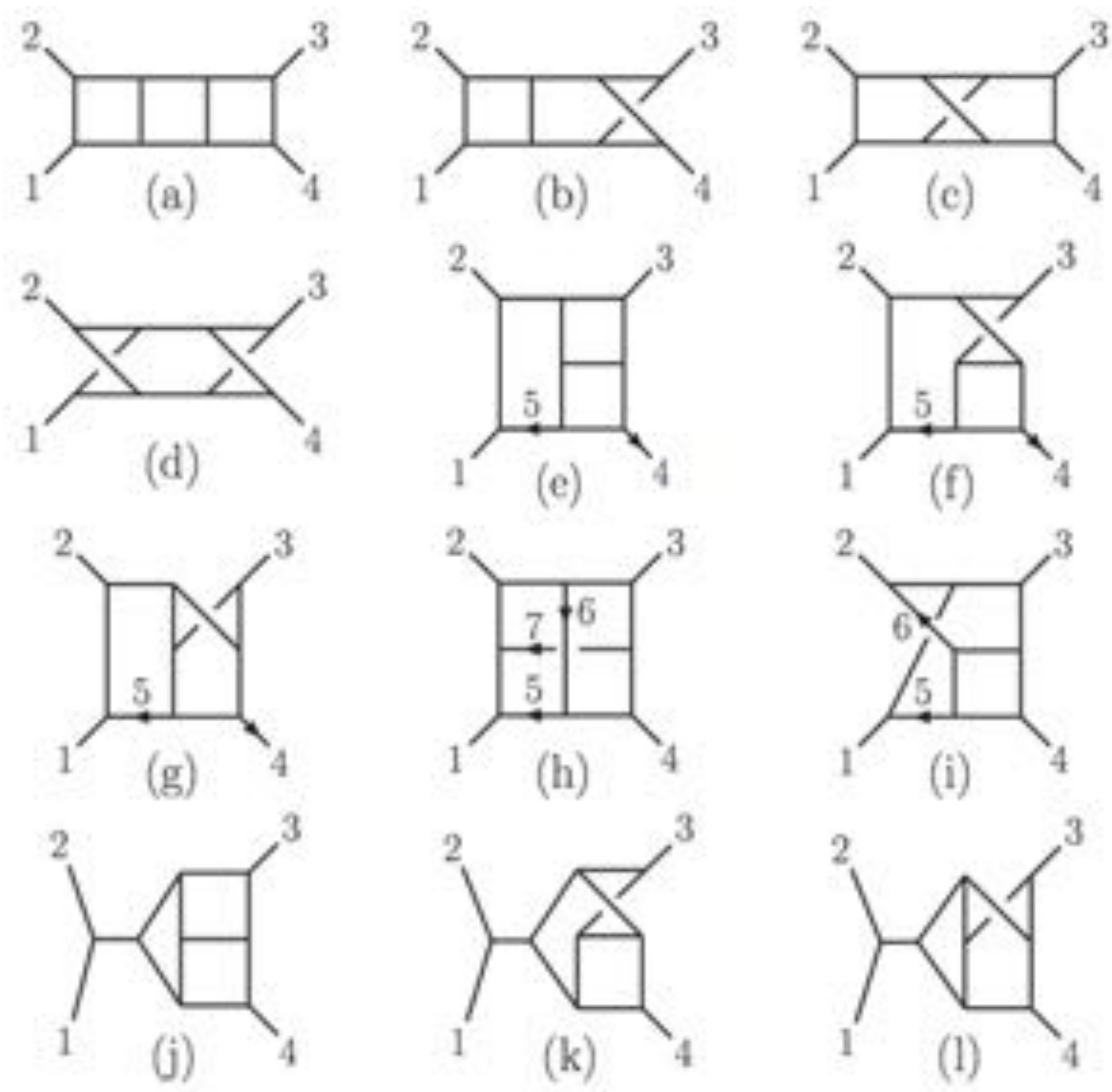


Consequence of unitarity: double copy structure holds.

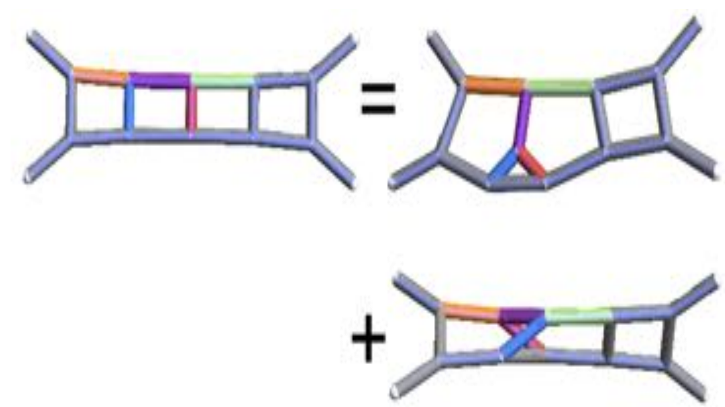
$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

Bern, JJMC, Johansson ('08,'10)

Calculate by Exploiting Color-Kinematics Duality



Leads to important constraints at tree & loop-level for gauge theories



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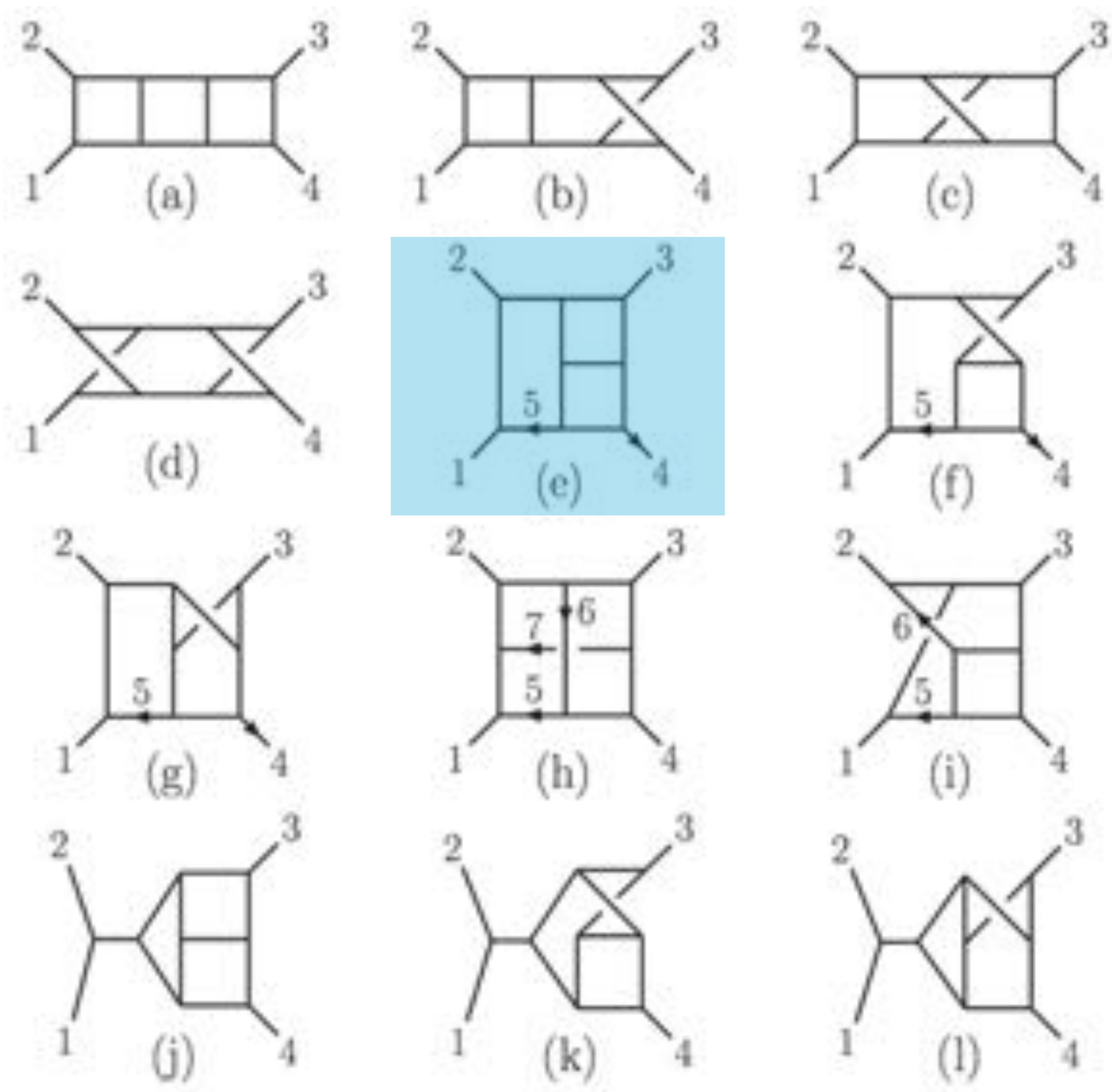

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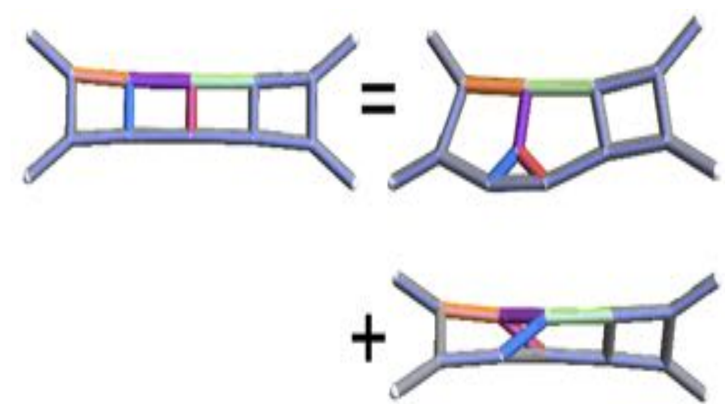
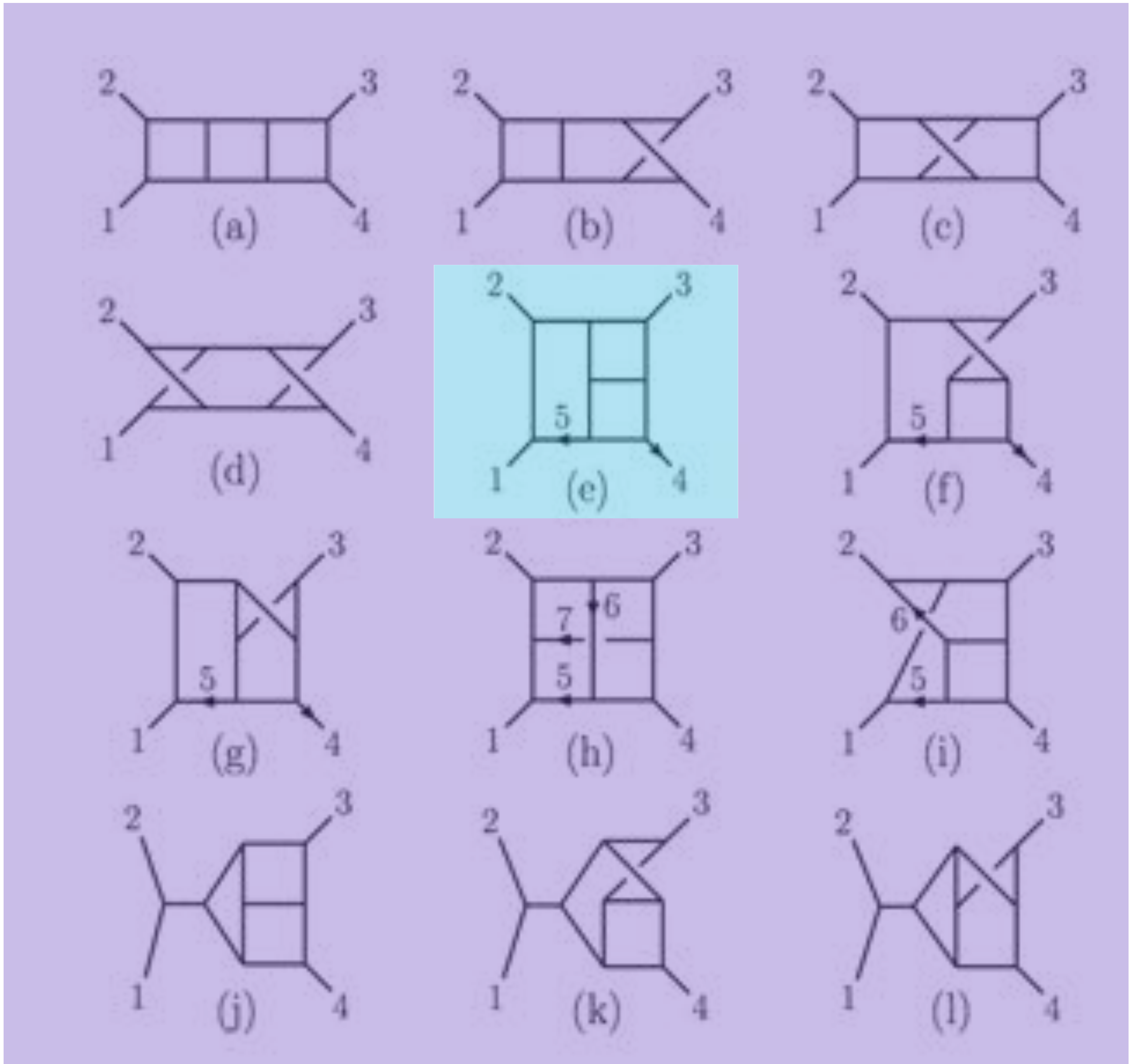


Bern, JJMC, Johansson ('08,'10)

Calculate by Exploiting Color-Kinematics Duality



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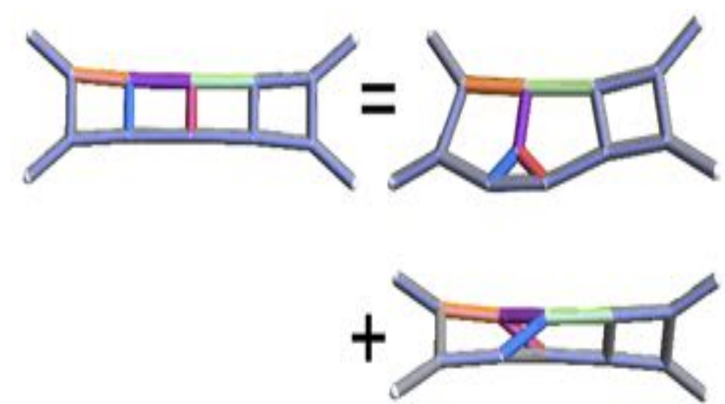
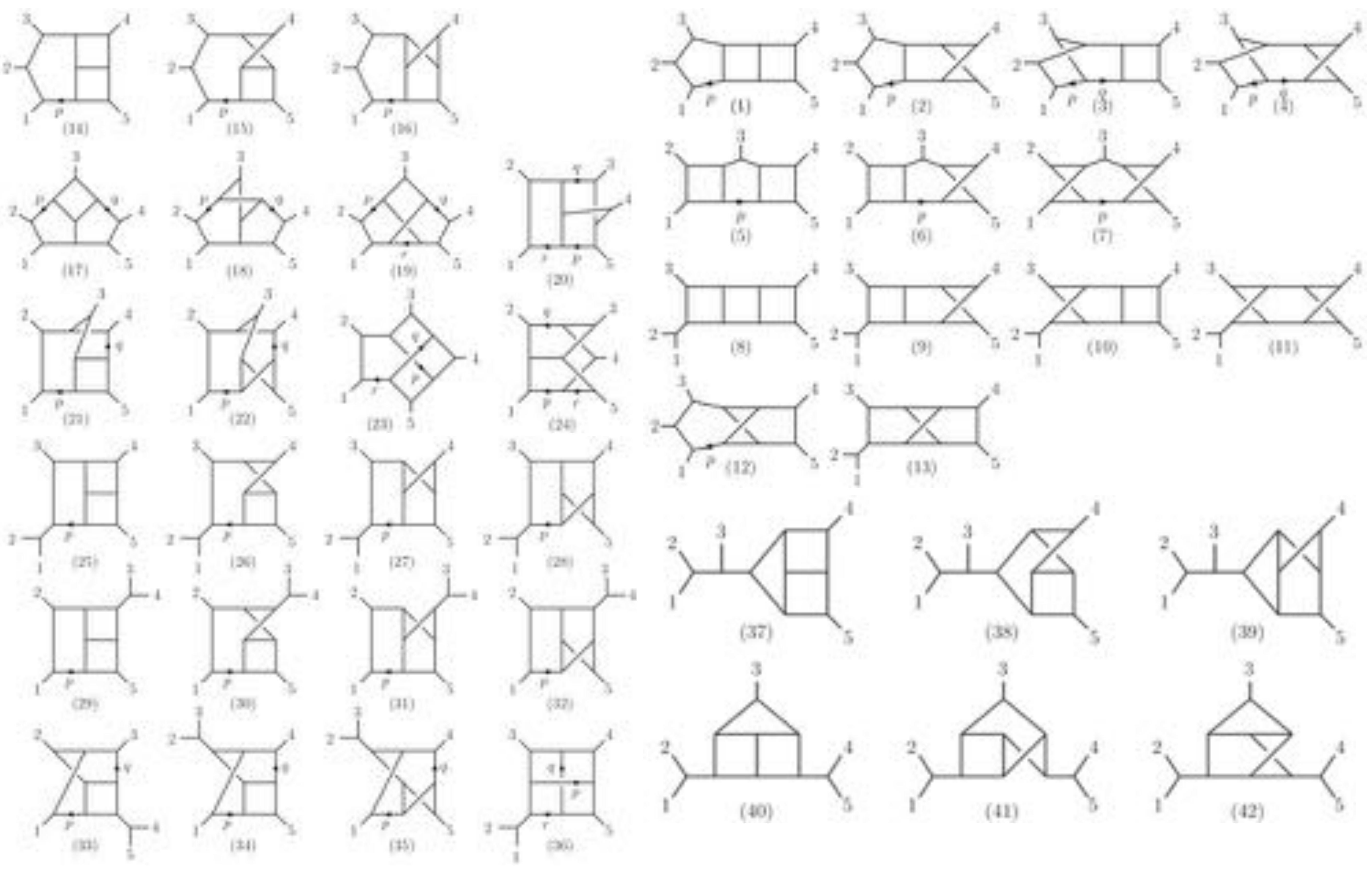


Gluons for (almost) nothing... gravitons for free!

Five point 3-loop N=4 SYM & N=8 SUGRA

JJMC, Johansson (to appear)



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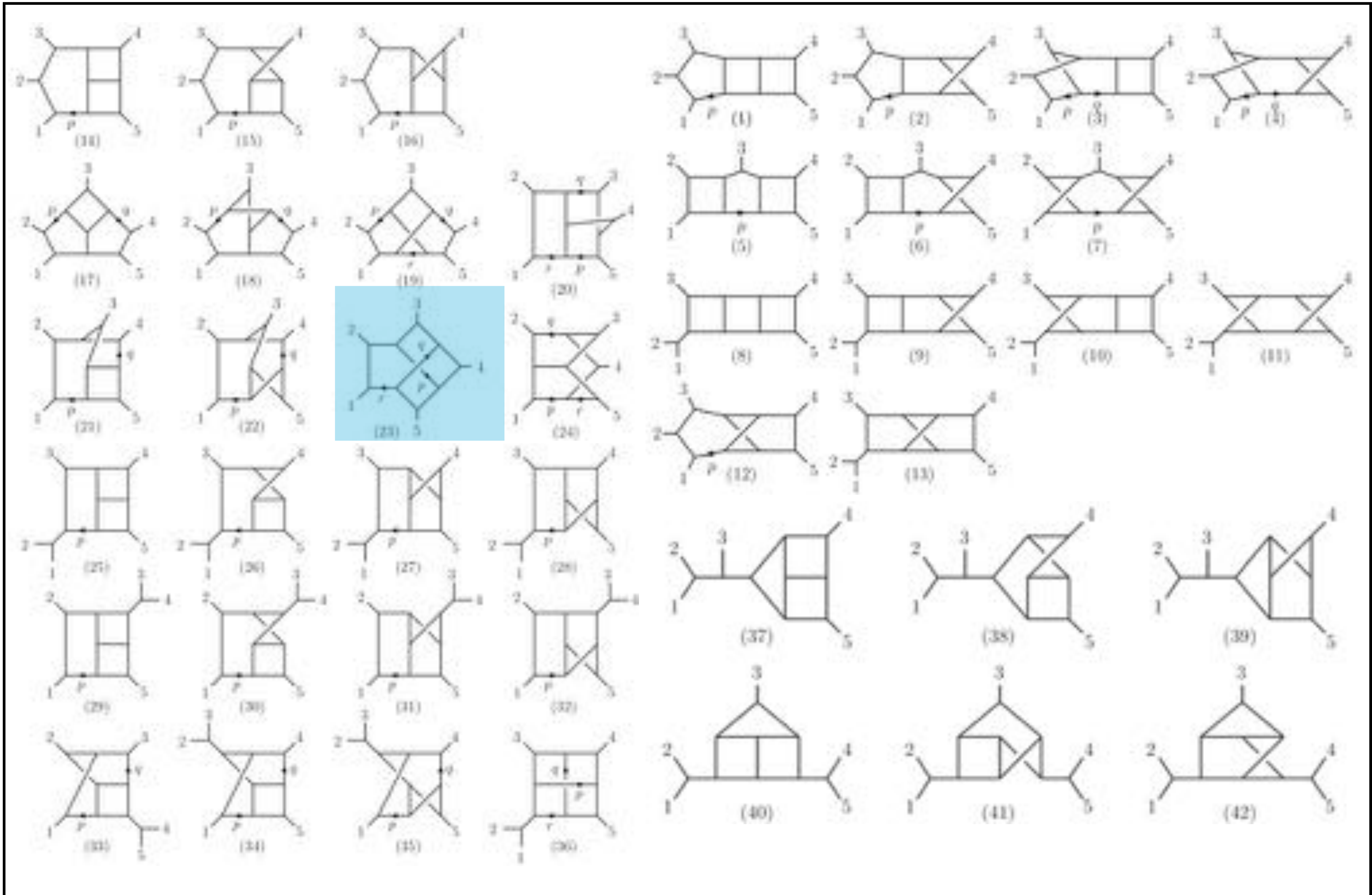

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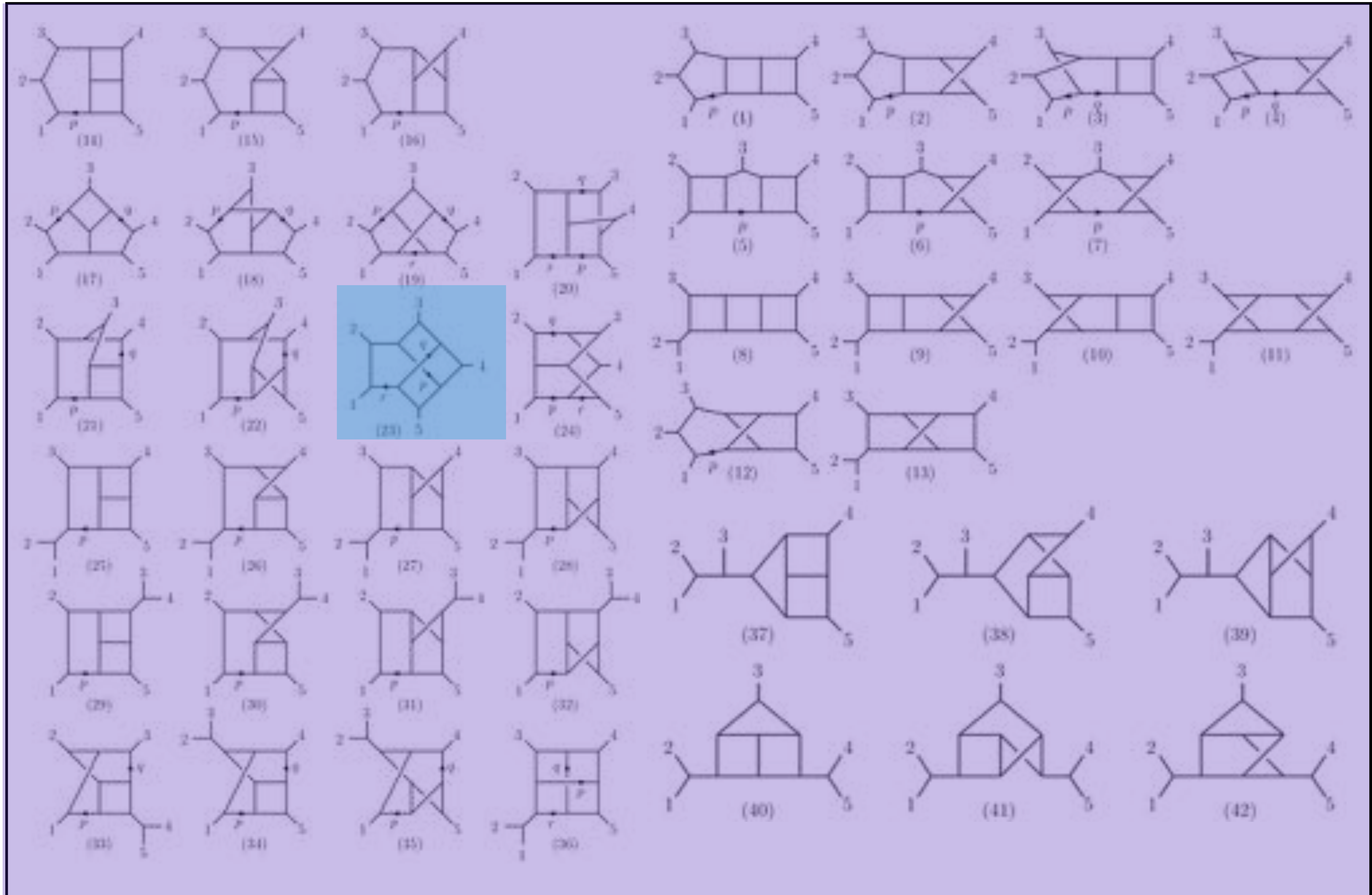
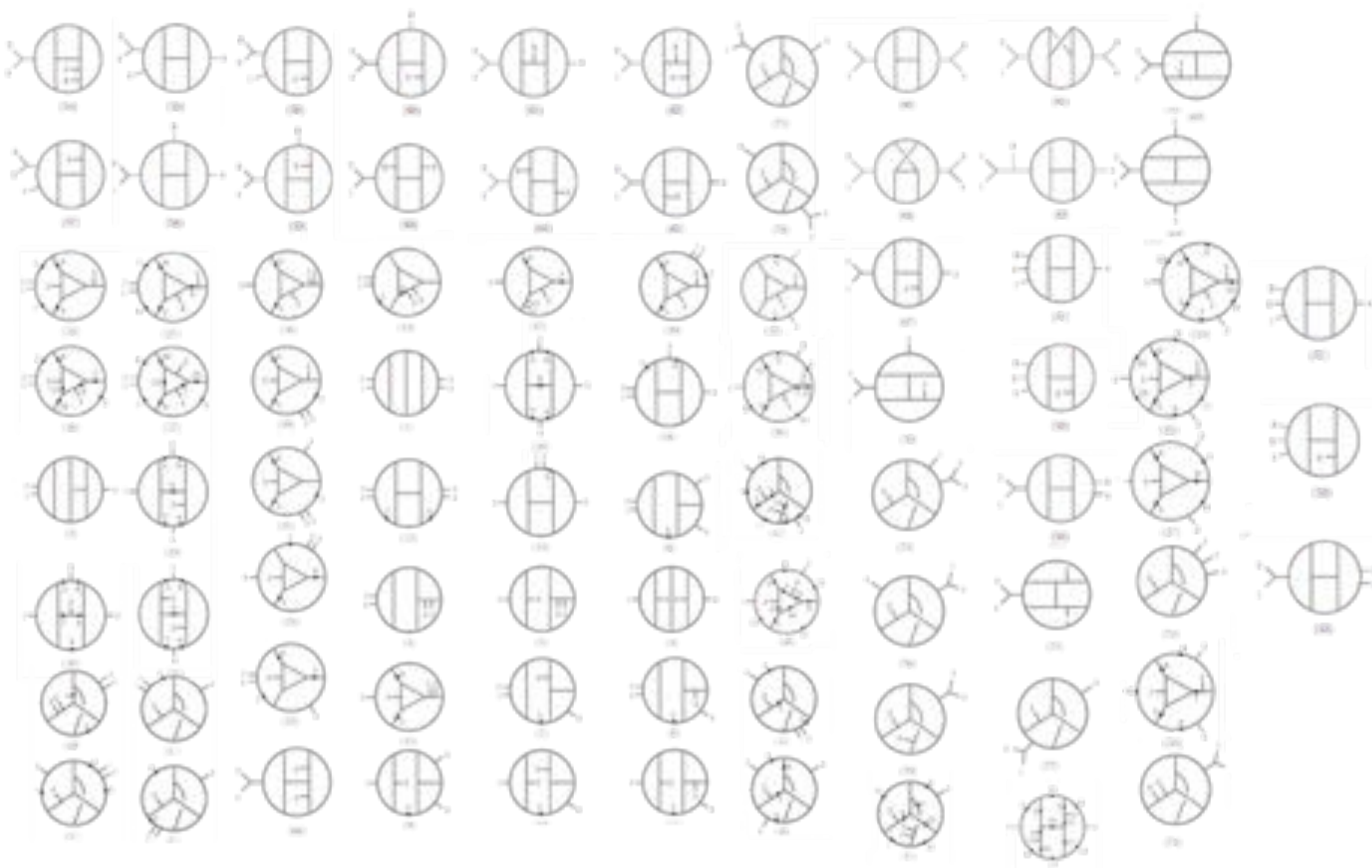


Full four loop N=4 SYM & N=8 SUGRA

Bern, JJMC, Dixon, Johansson, Roiban (2012)



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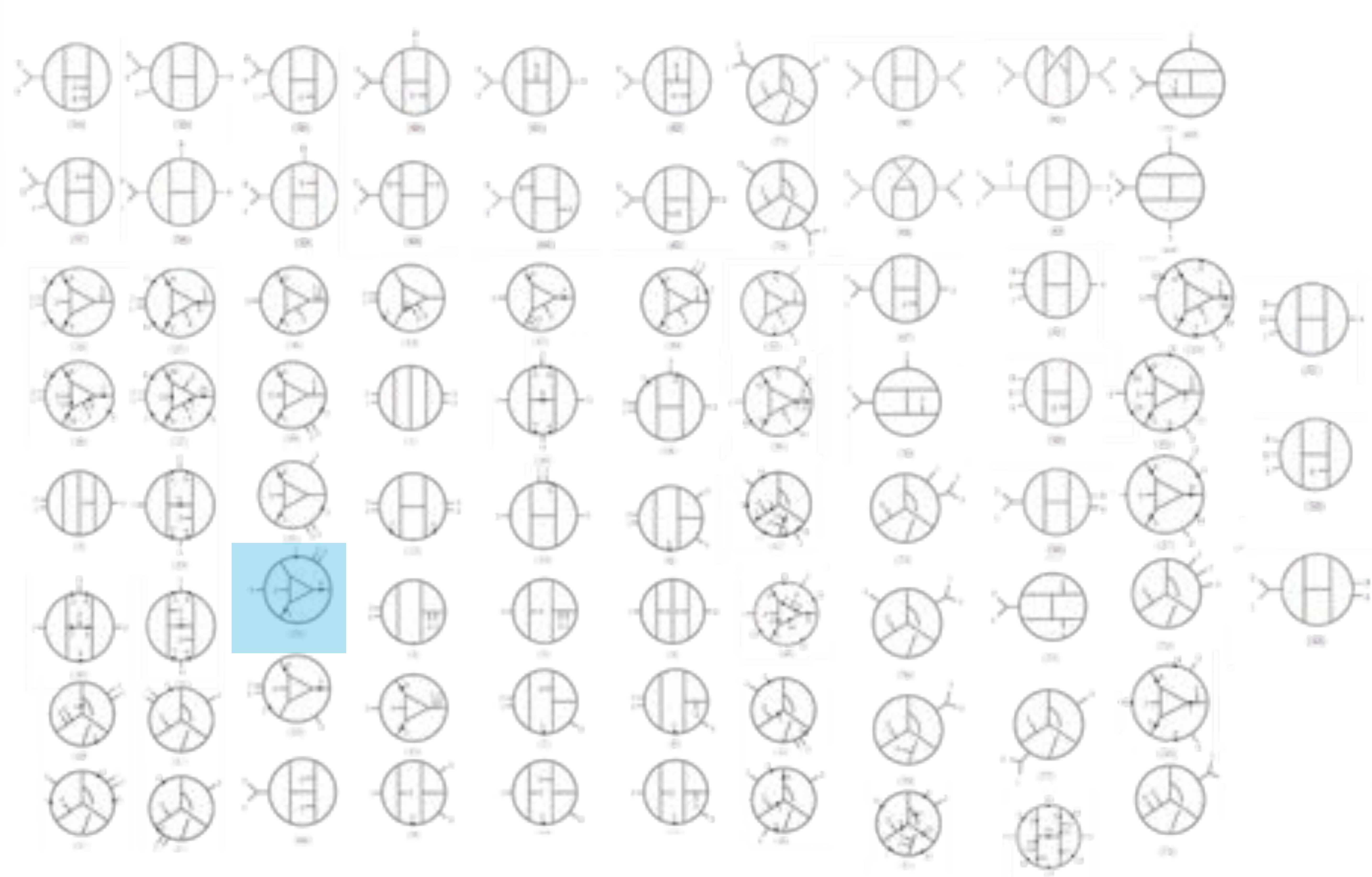

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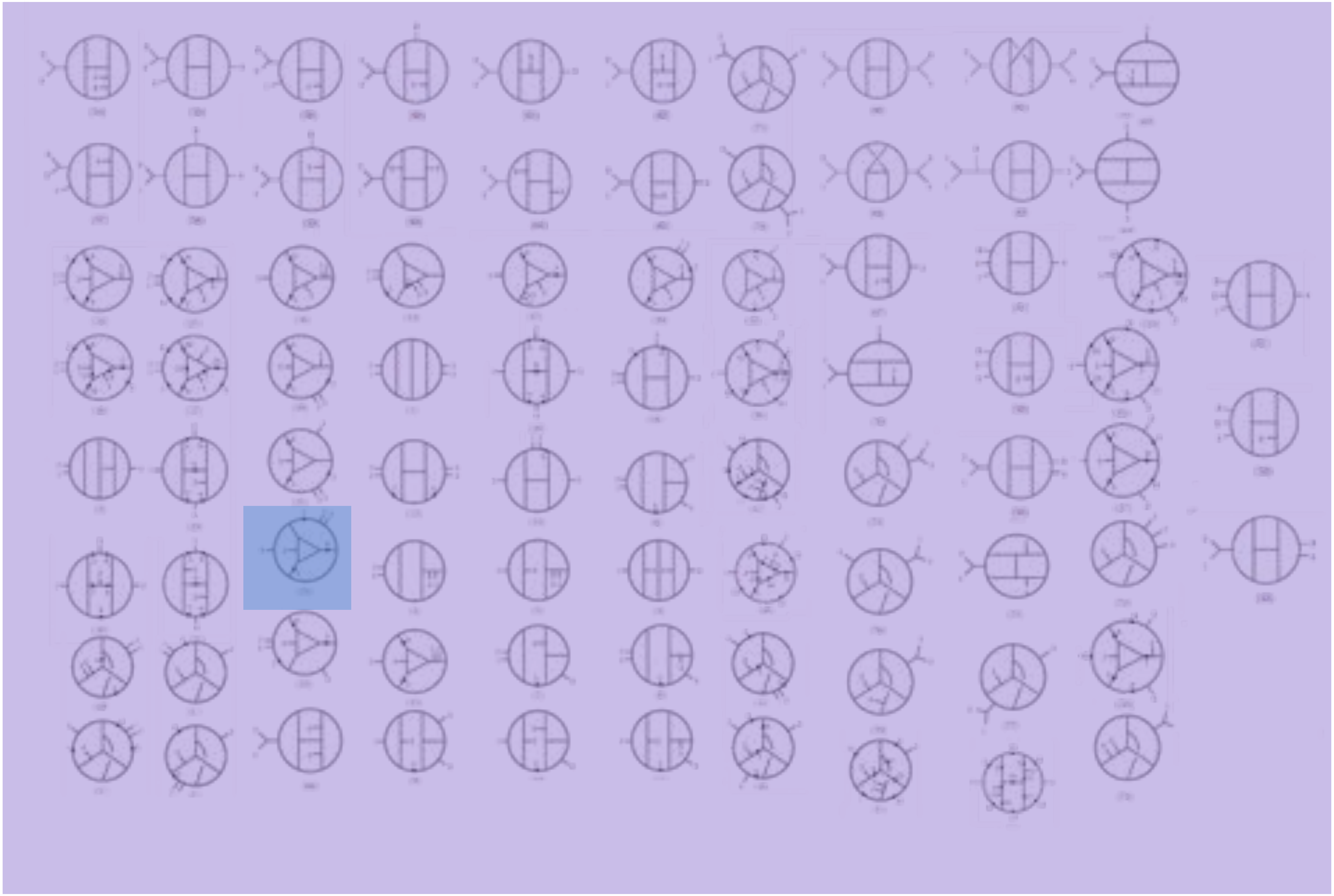
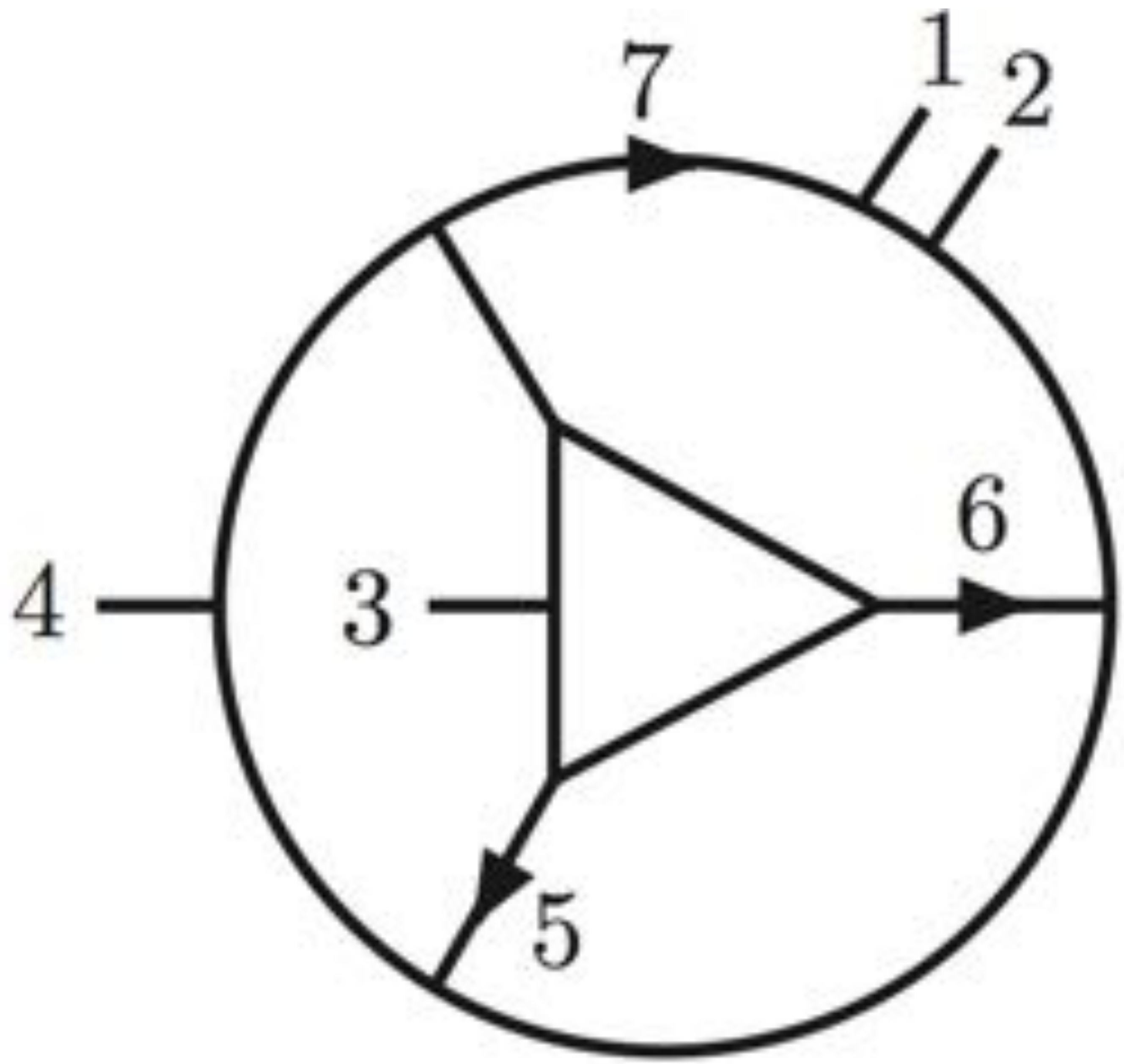


4-loops Maximal SUSY



Many things to be learned, not the least, the existence of integral relations between gauge and gravity theories

Bern, JJMC, Dixon, Johansson, Roiban (2012)

Problem Solved?

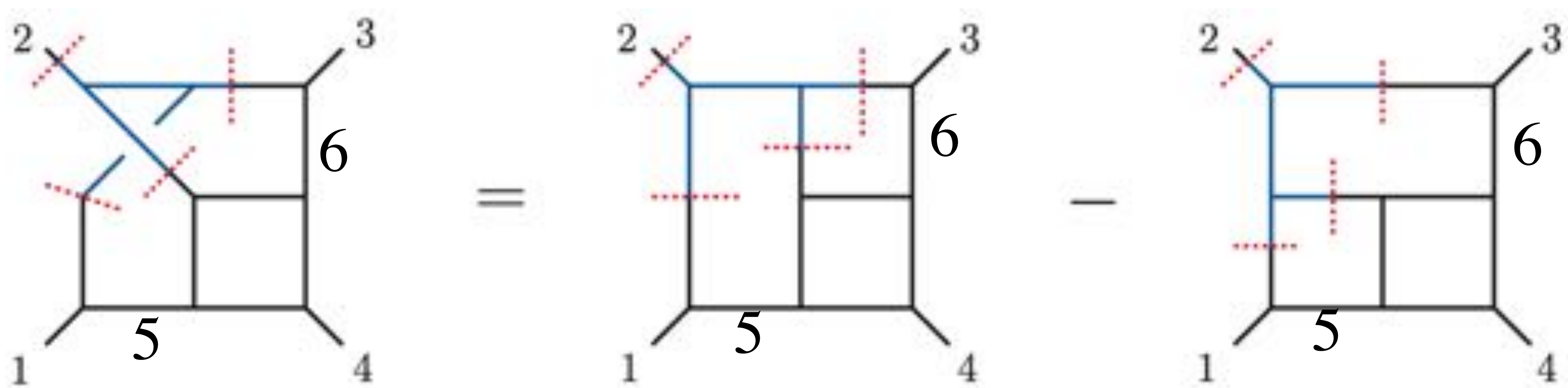
No.

We want all-order understanding!

What's the barrier?

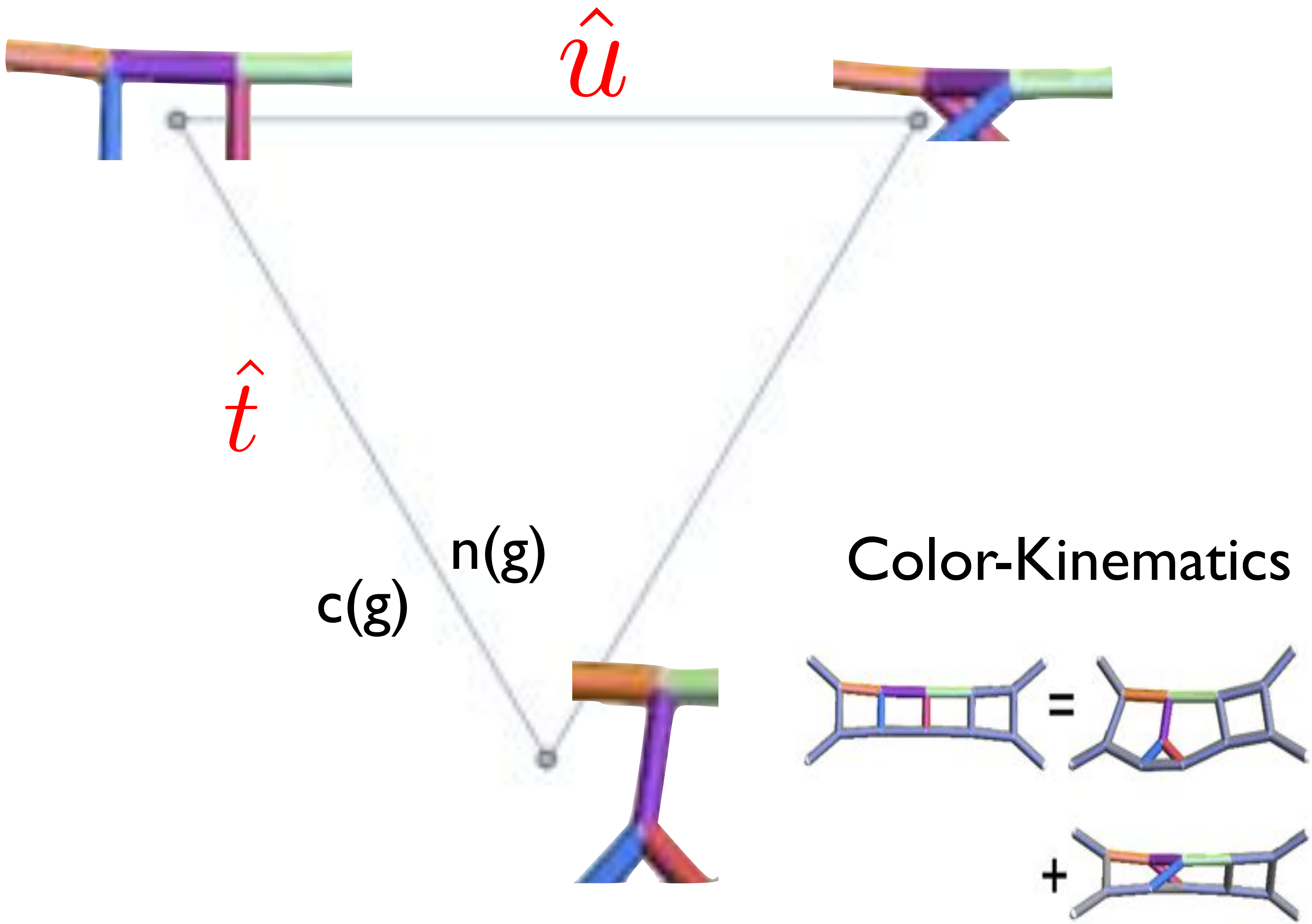
Frustrating Problem:

- Exploiting Color-Kinematics duality at loop-level means solving functional equations: number of master graphs controlled, but require a parameterized ansatz.

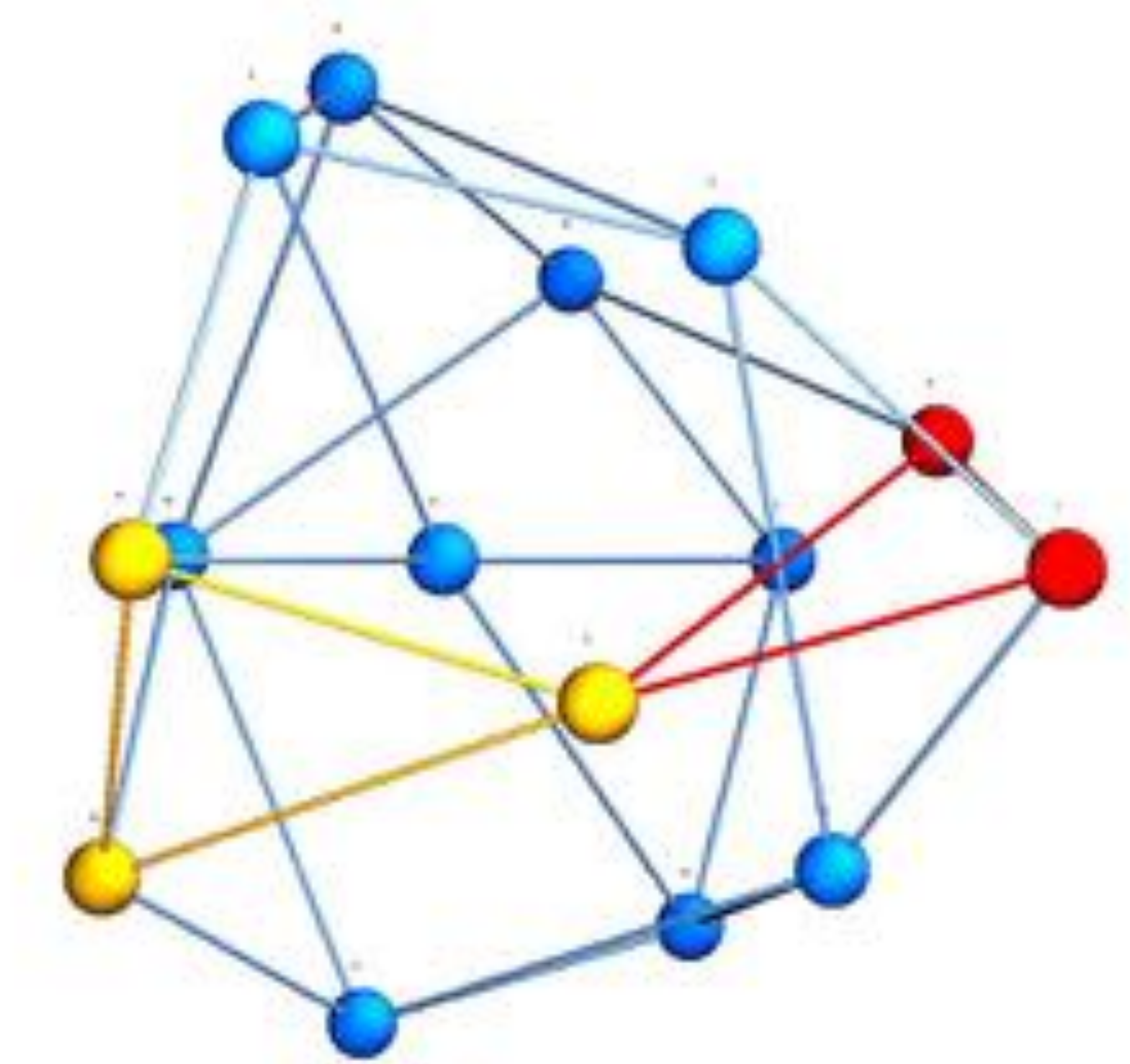


The set of multi loop Jacobi equations will relate the ***same*** numerator functions with permuted arguments.

Convenient language: graphs of graphs

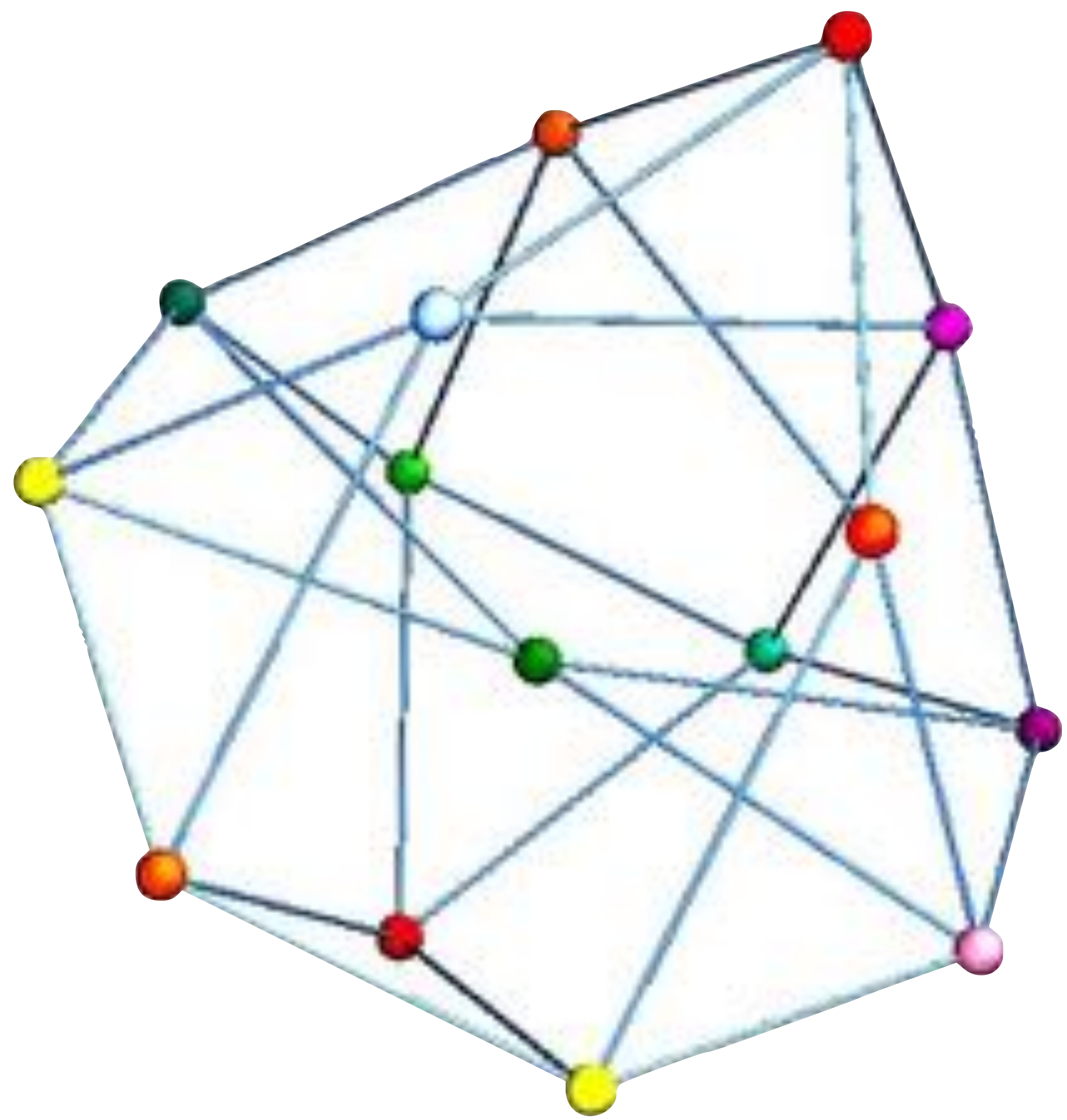


tree-level, imposing CK is no problem

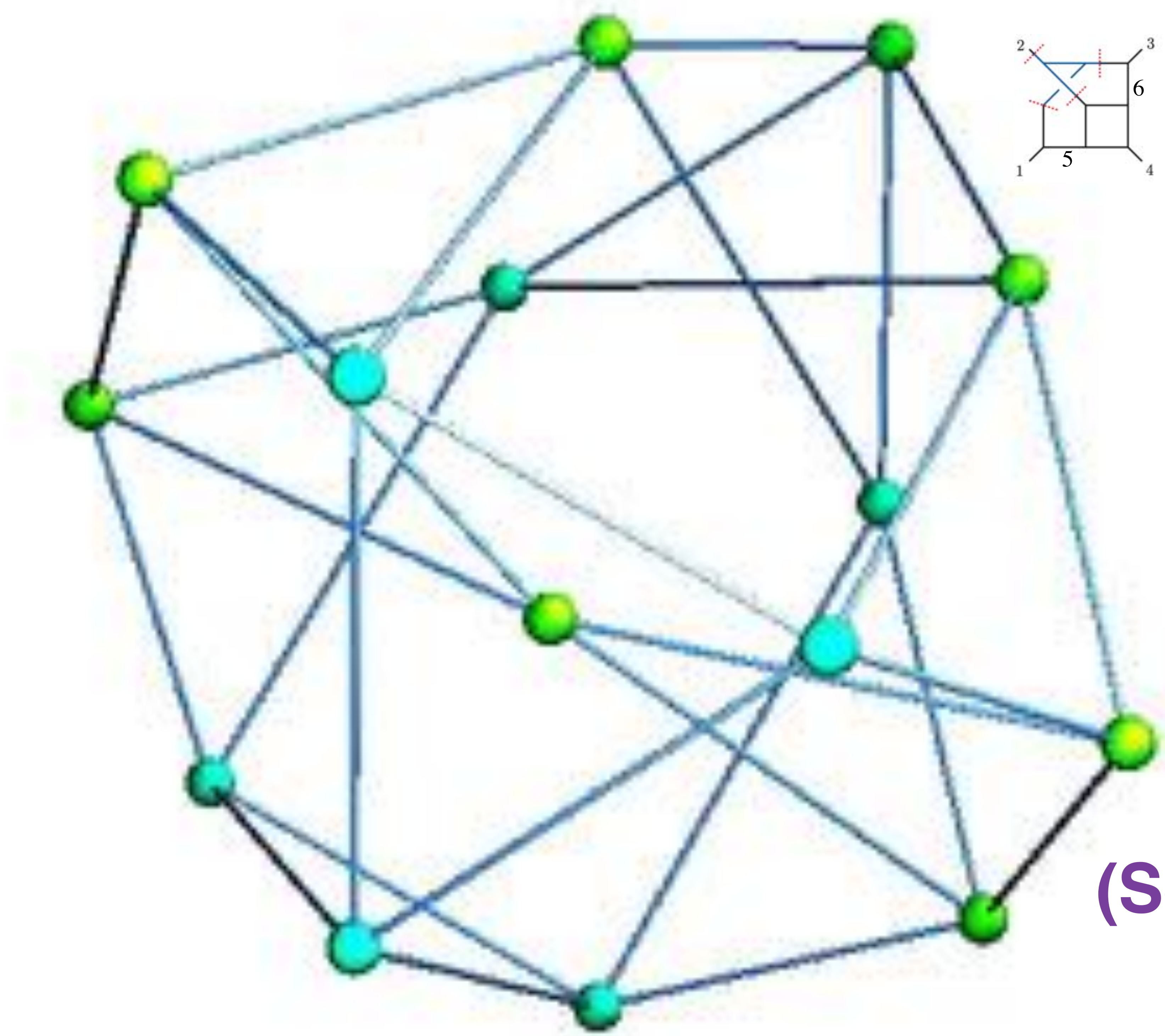


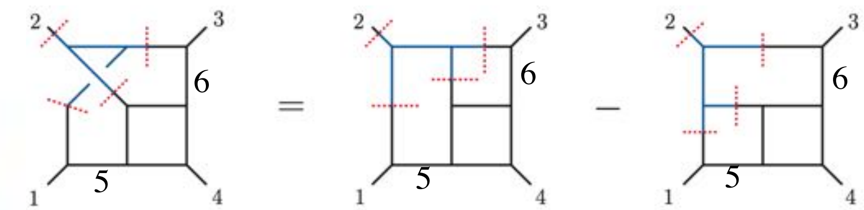
each vertex is a graph

each triangle represents a Jacobi identity between graphs tree-level, imposing CK is no problem



as each node represents a separate graph, Jacobi eqns impose linear relations between numerators loop level, functional constraints





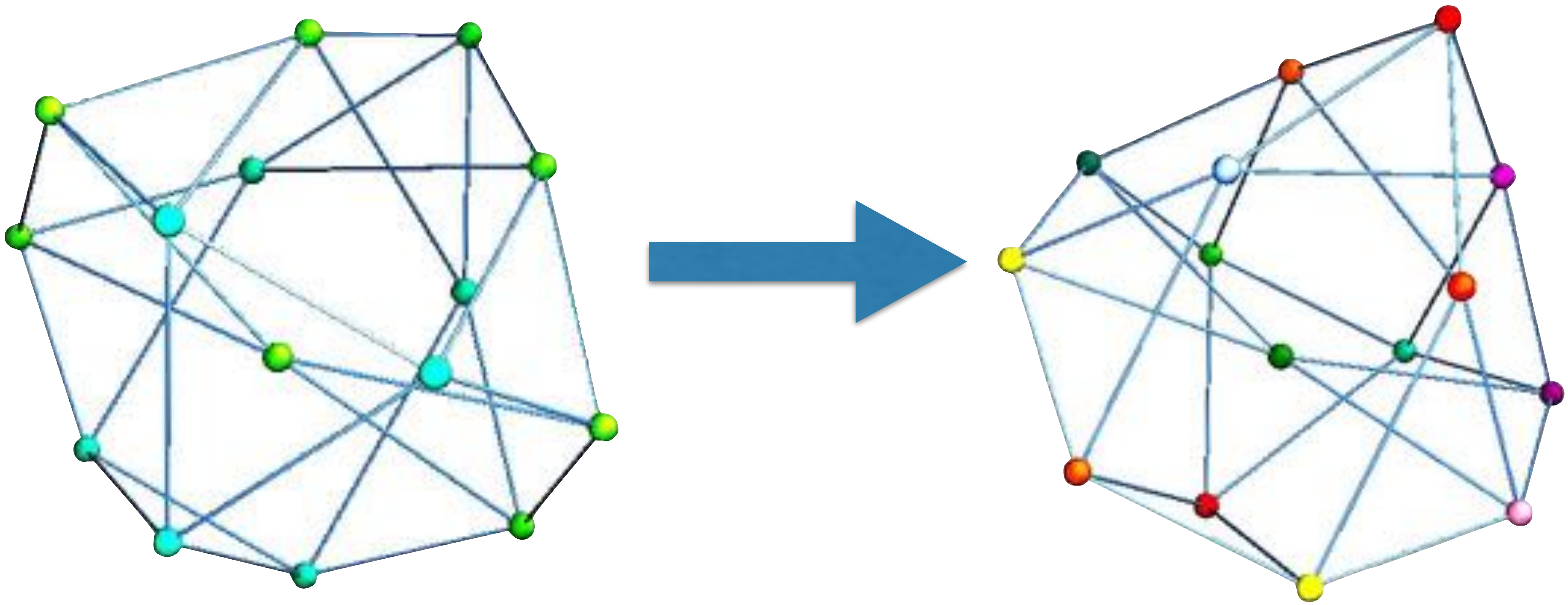
LABEL-
SHIFTING
PROBLEM

(See also Jara's talk)

nodes can be the same graph topology but with permuted labels!

Off-shell pre-Integrand:

a solution to label-shifting



introduce a distinct graph for every possible labeling of m-point L-loop graph topologies with L indep momenta.

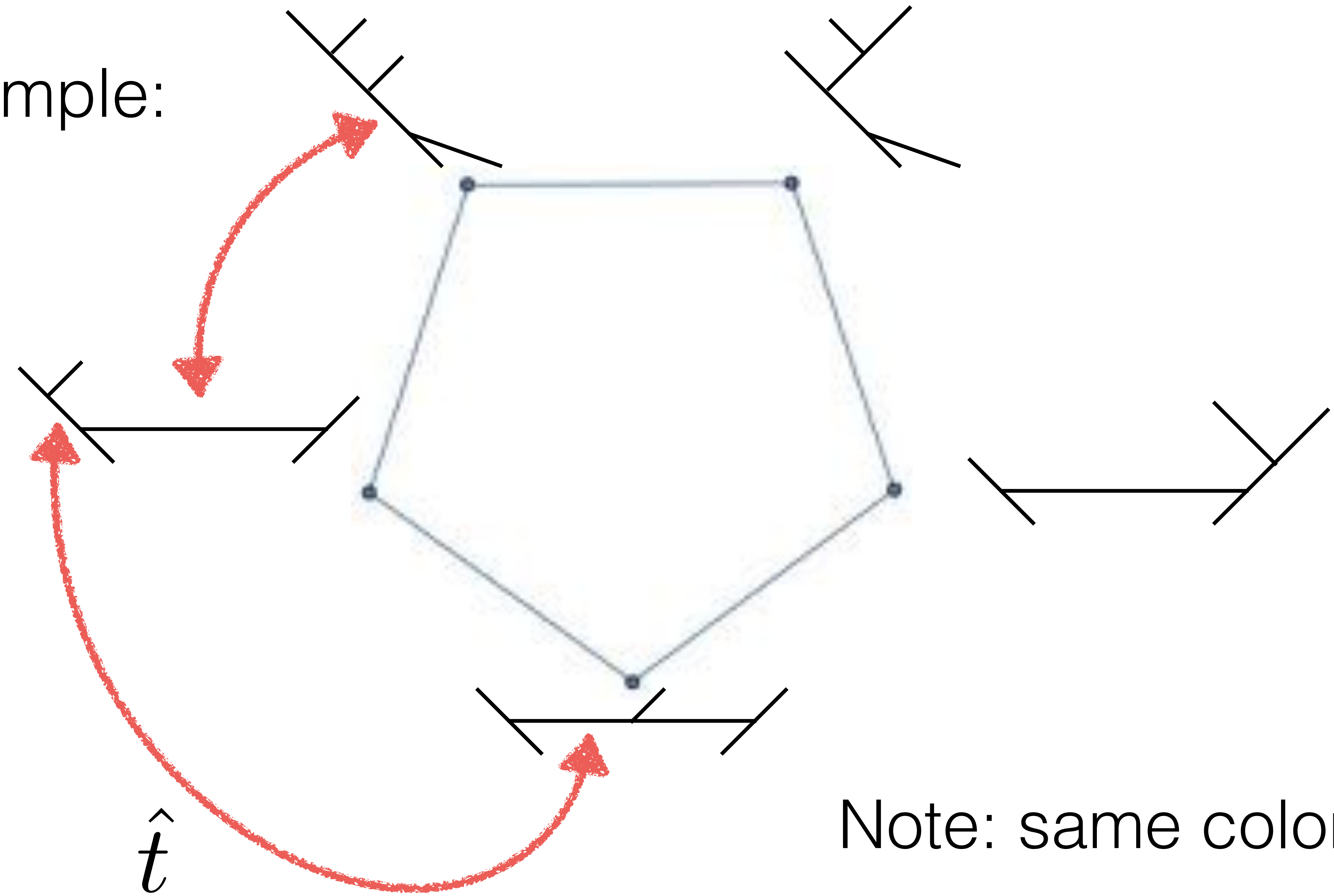
this will be isomorphic to a subset of (2L+m)-point tree graphs, with 2L “ext” labels: $\{l_1, -l_1, \ldots, -l_L, l_L\}$

3. Exposing a geometry in the S-matrix

(the best polytopes are graphs of graphs!)

Graphs contributing to a color-ordered tree, generate the 1-skeleton of **Stasheff polytopes** joined only by $\hat{t}$

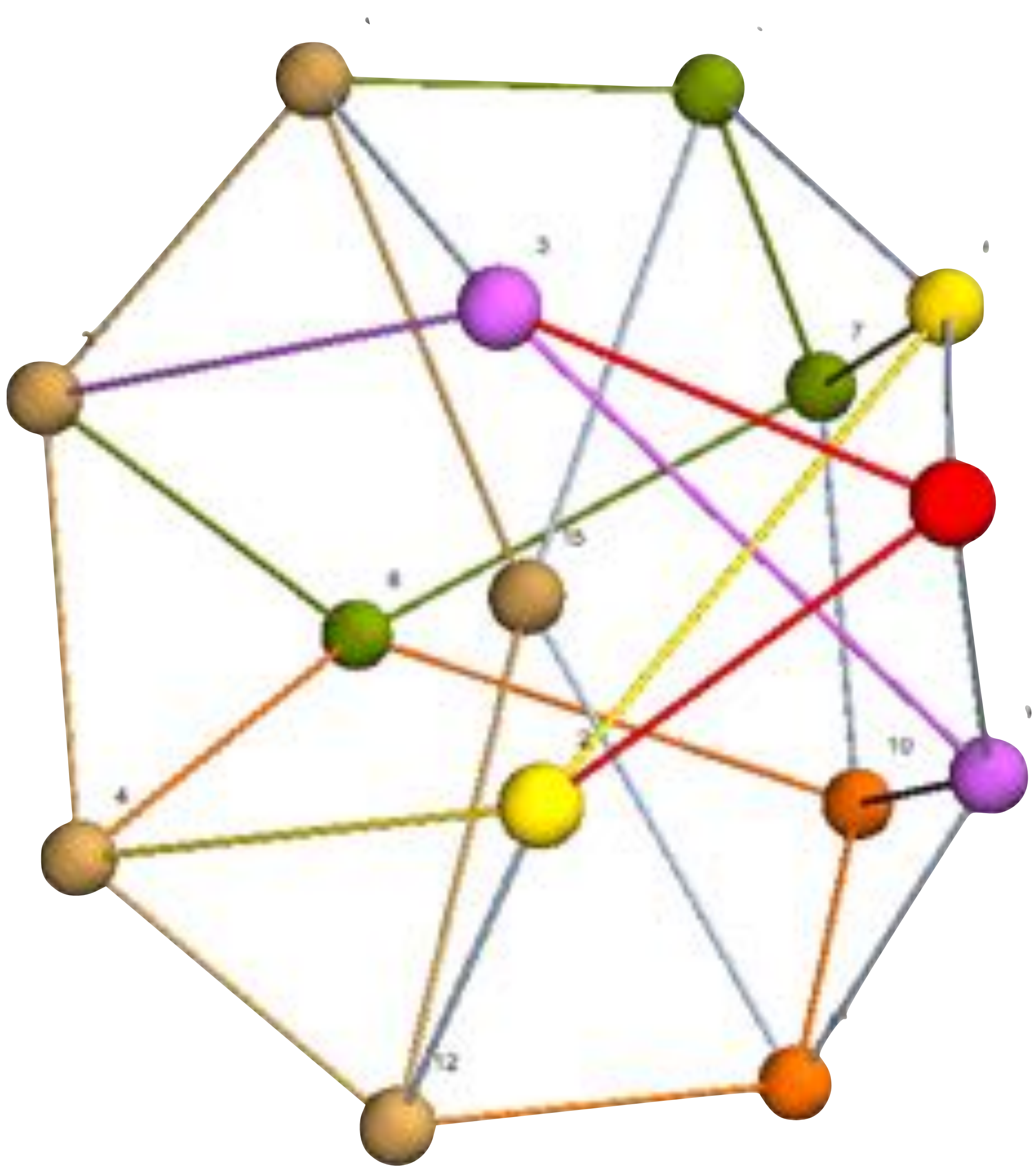
5pt example:

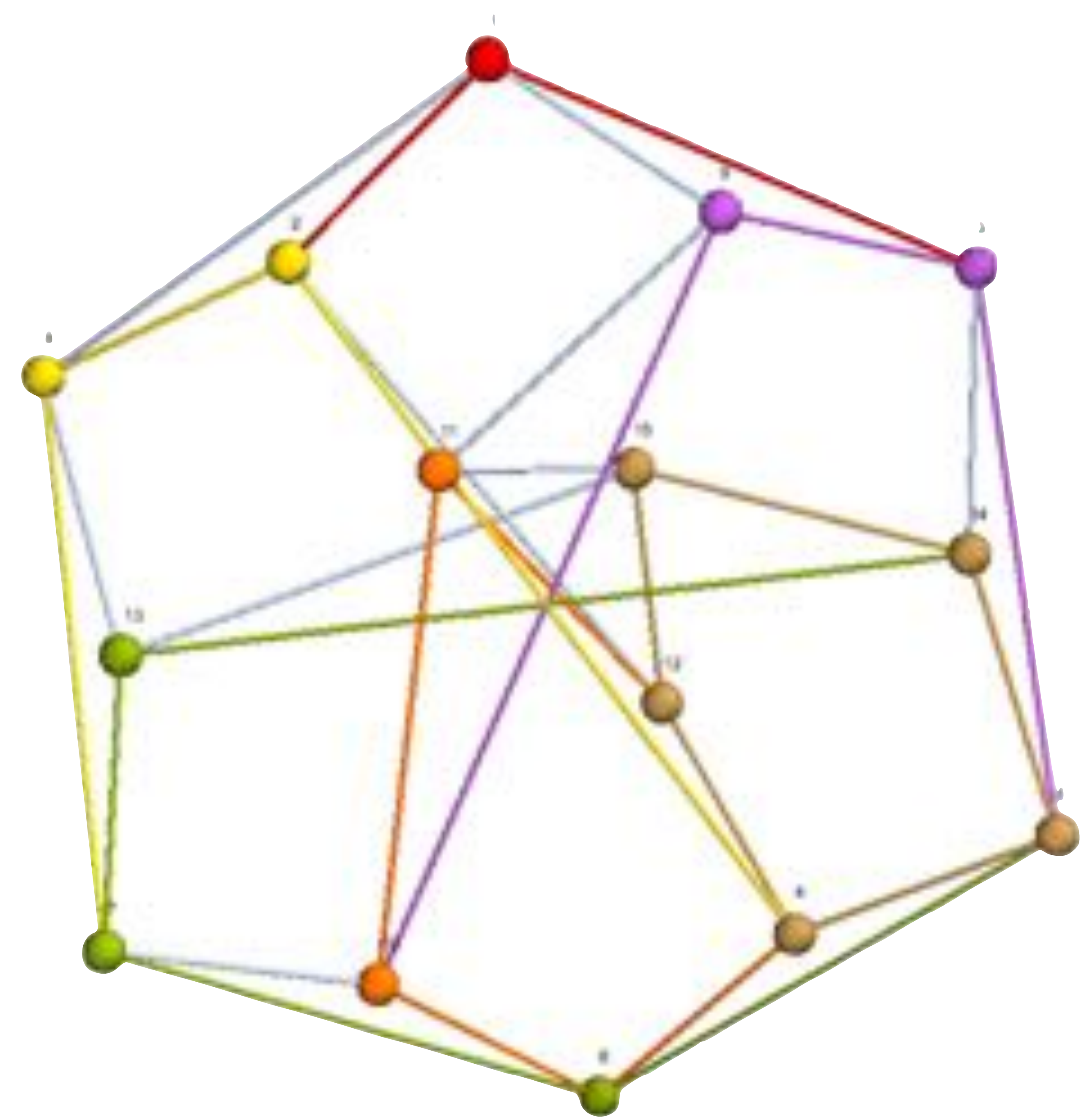


Note: same color-order!

(these polytopes are also called **associahedra**)

You might think you need (m-2)! of these color-ordered amplitudes to capture everything because this is what is required to touch every vertex at least once:





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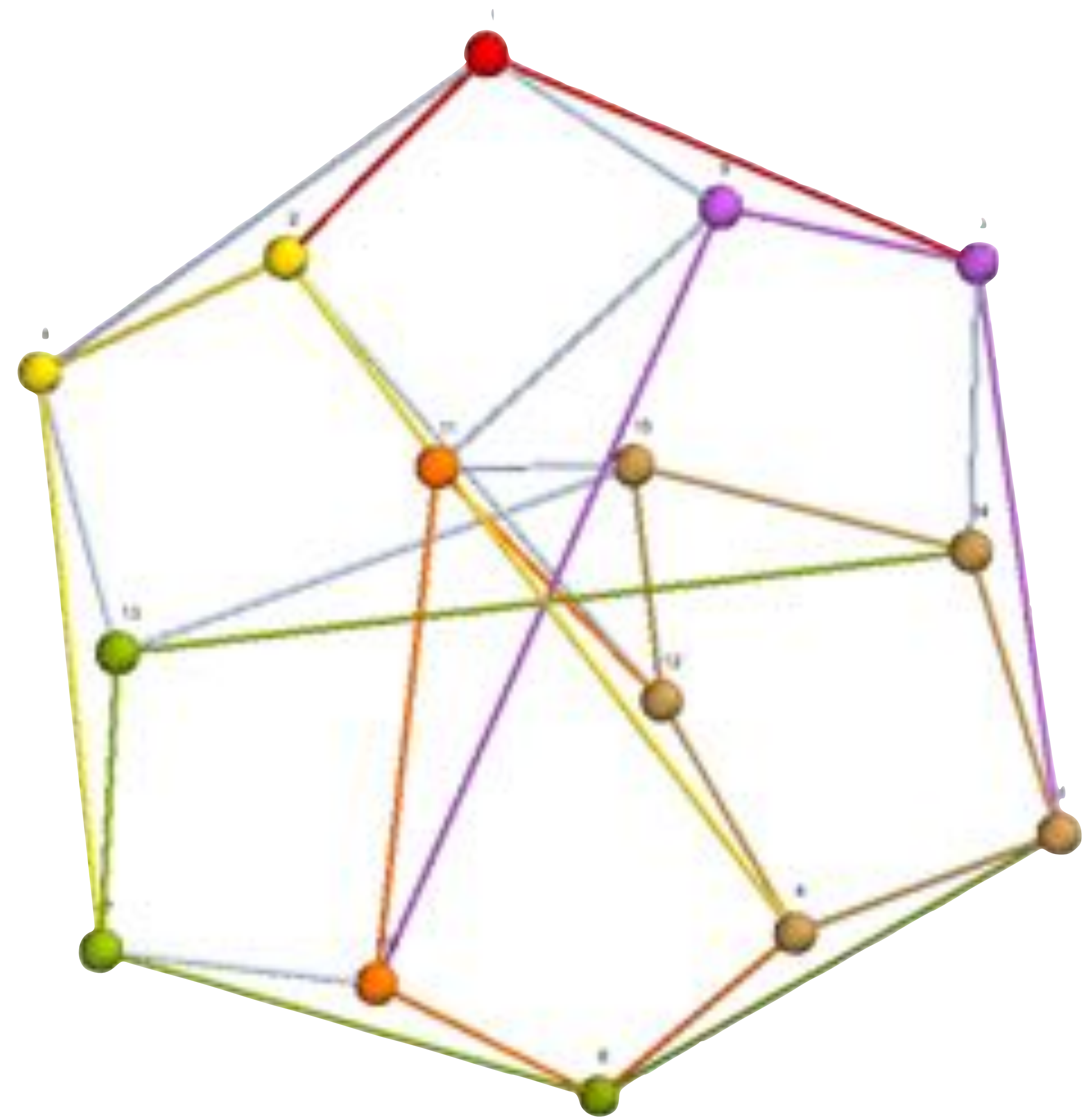
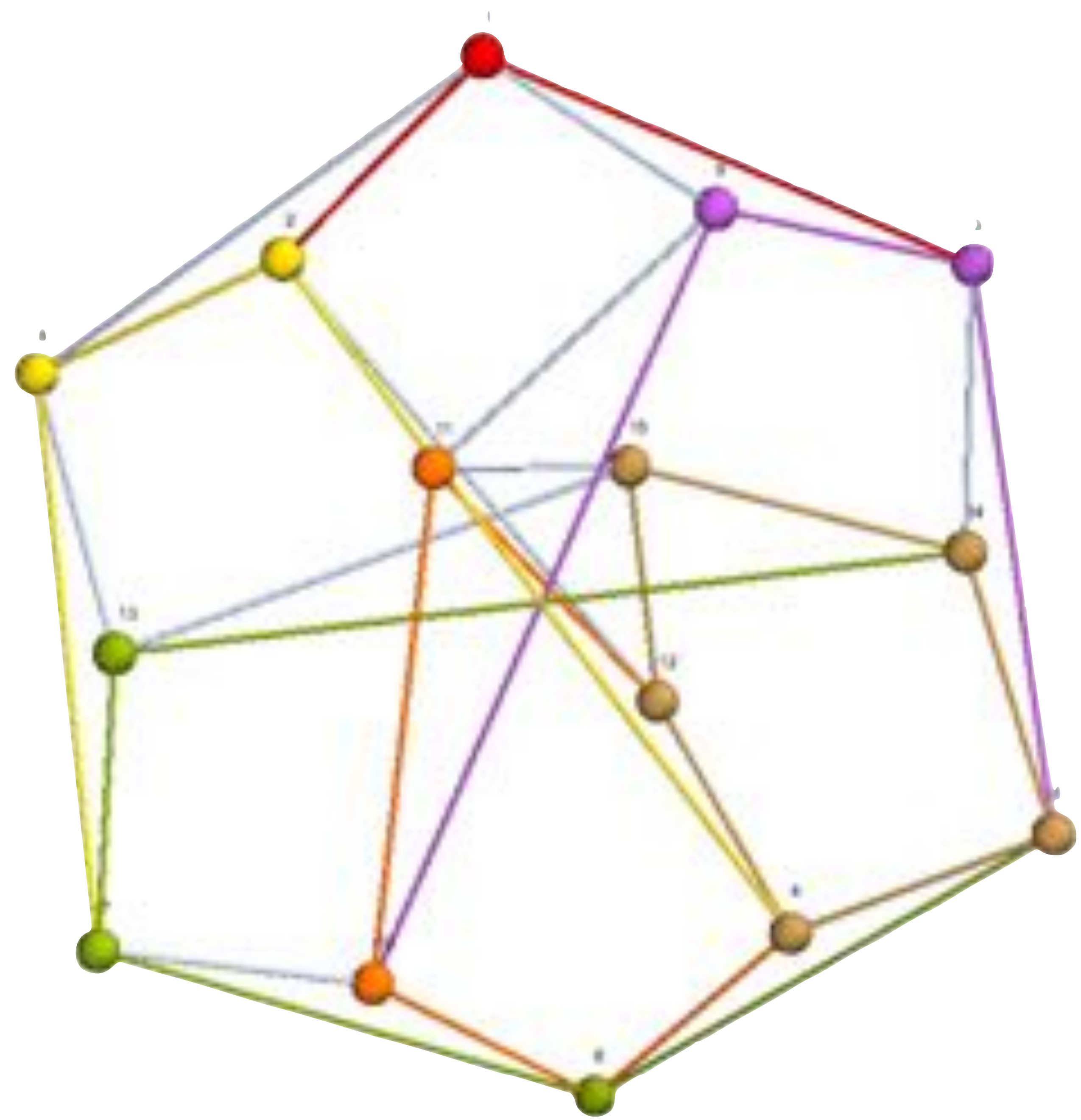


In fact, such a choice is the KK-basis, proven sufficient by Del Duca, Dixon, and Maltoni

But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone



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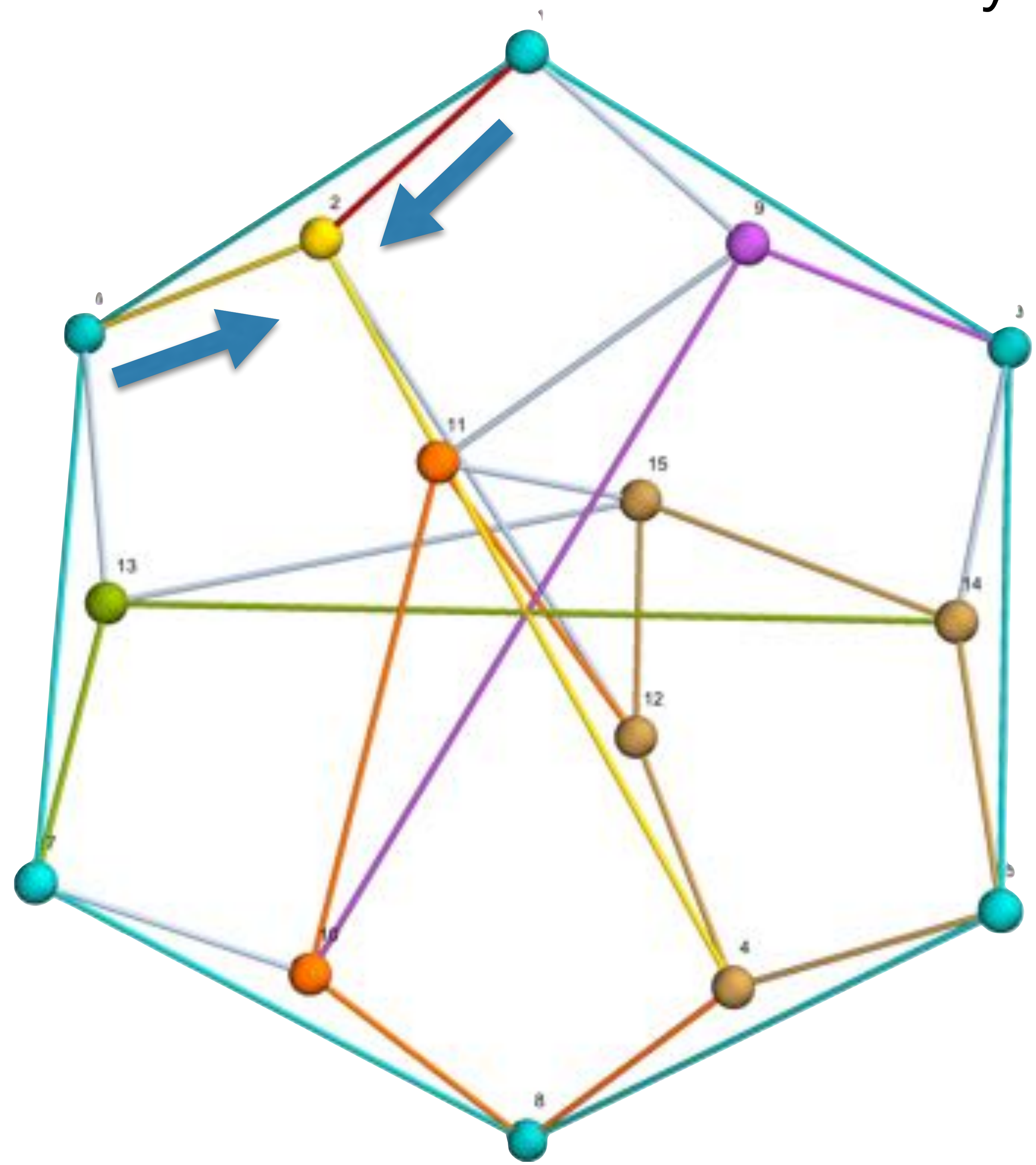

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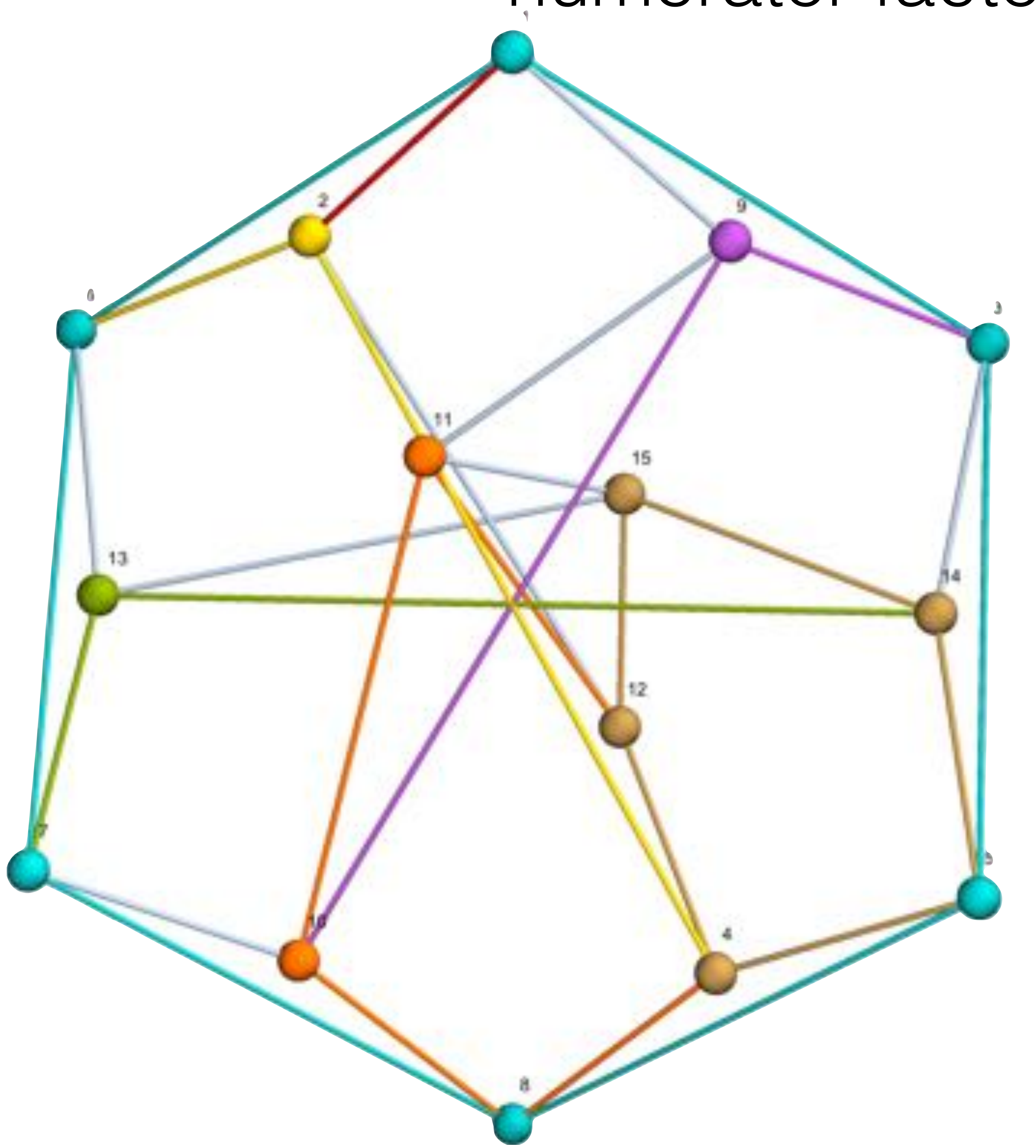
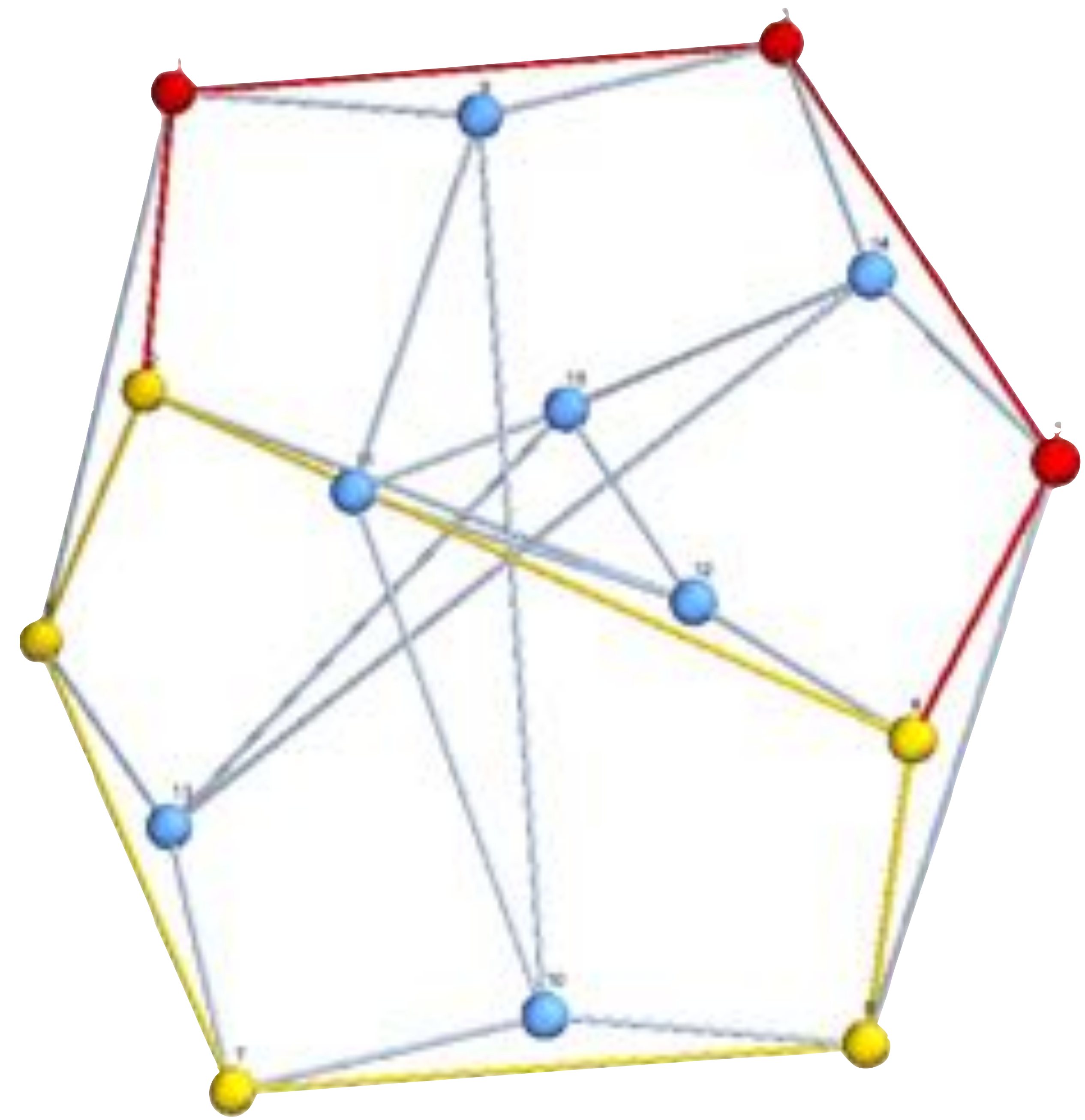
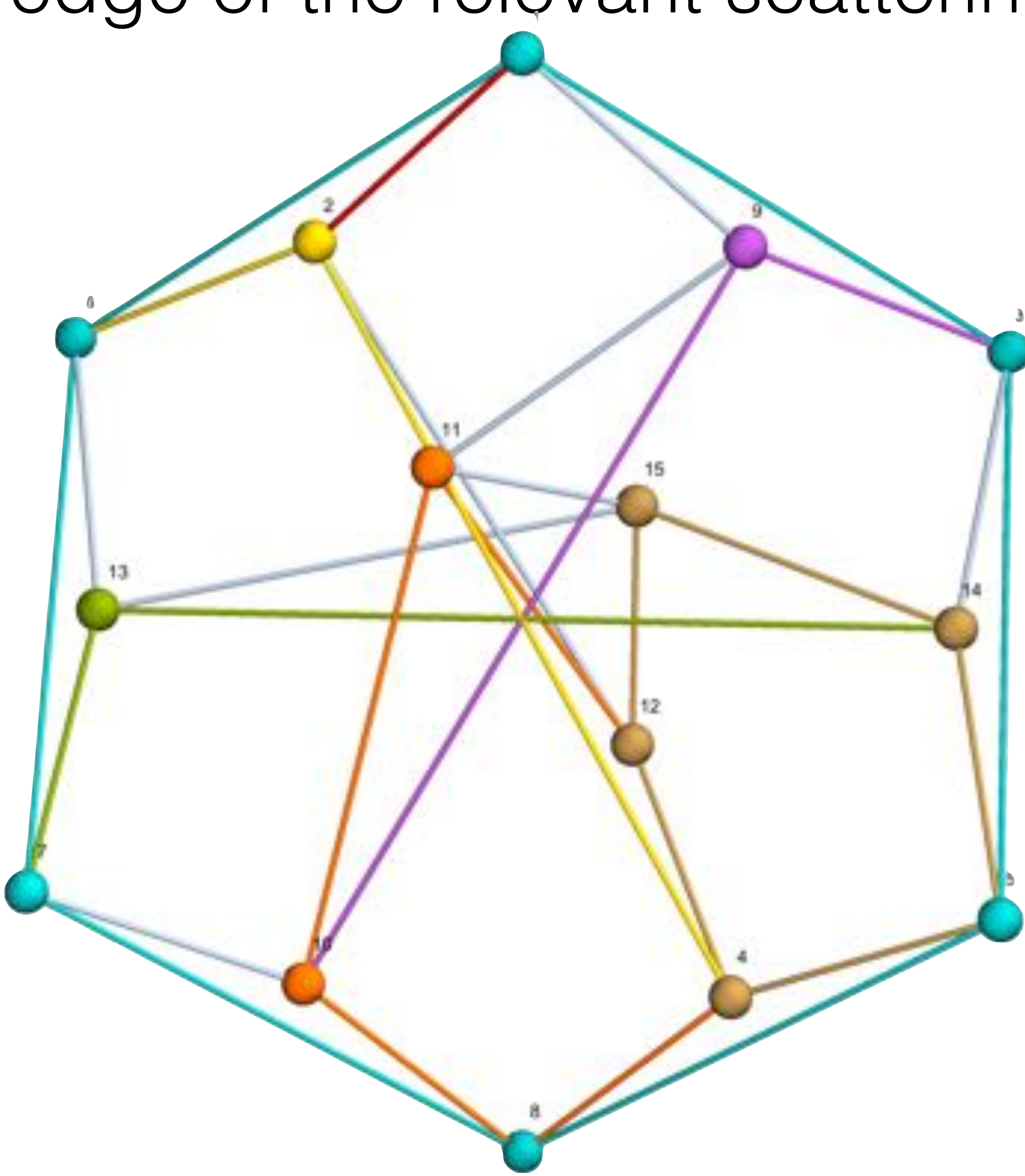
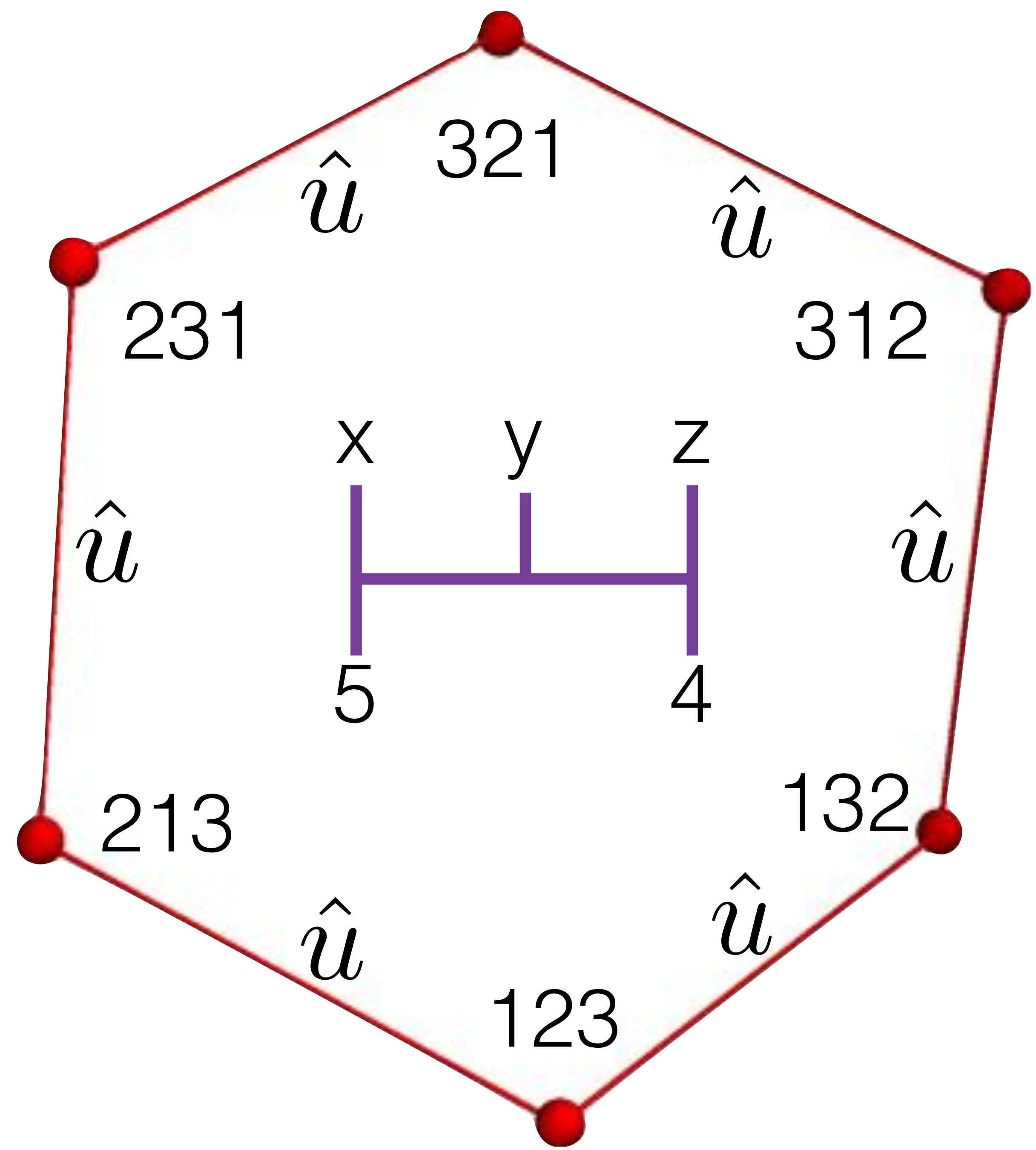




This reduces the set of necessary color-ordered amplitudes (associahedra) to (m-3!)

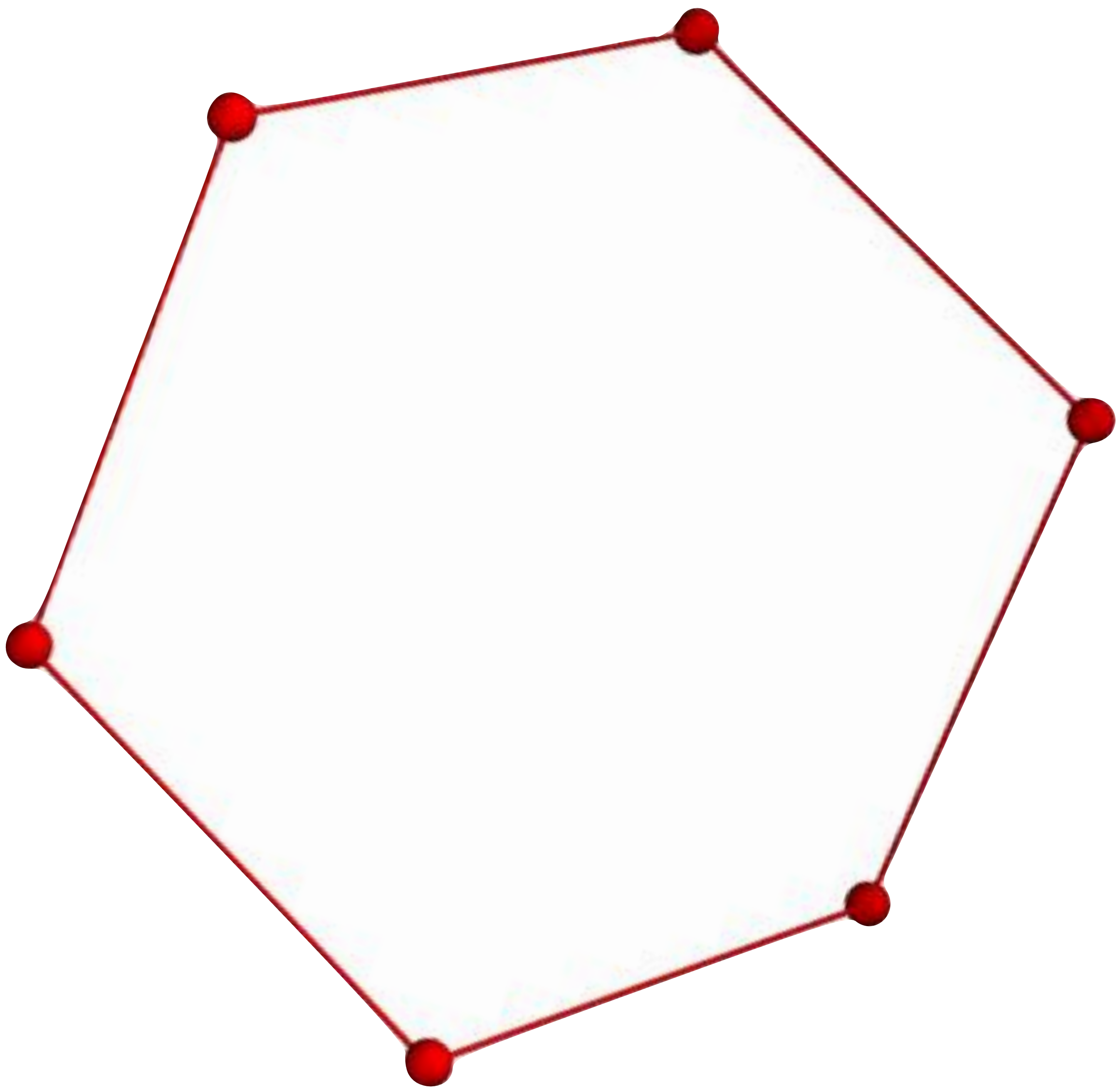
At every multiplicity the masters can be chosen to form the 1-skeleton of a polytope related by $\hat{u}$ on every internal edge of the relevant scattering graphs

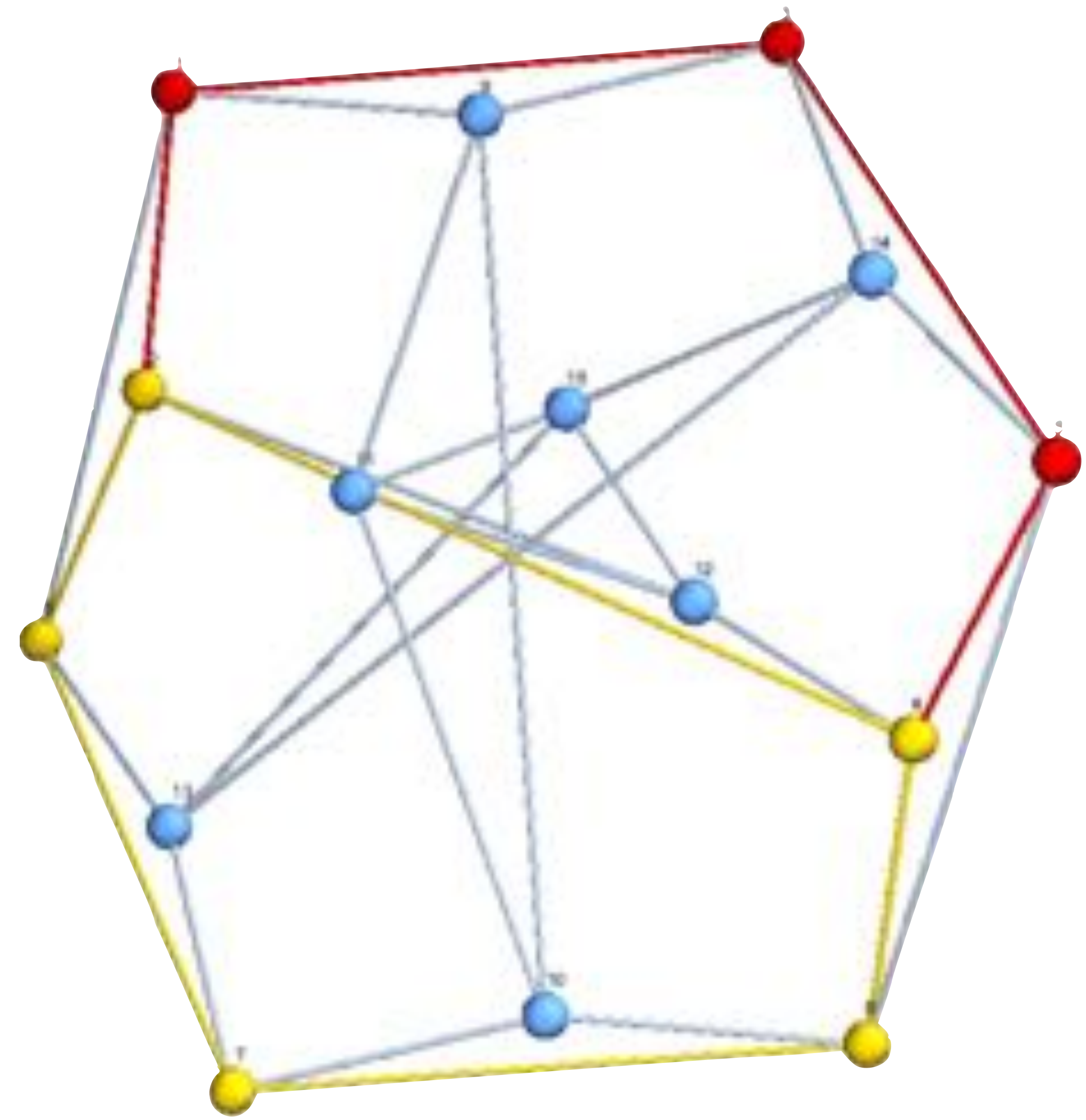




(these polytopes are called **permutahedra**)

Can linearly solve for the (m-2)! numerators of the masters in terms of the (m-3)! “BCJ” independent color-ordered amplitudes. In fact you get (m-3)! numerators in terms of the color-ordered amplitudes and (m-3)(m-3)! free functions.

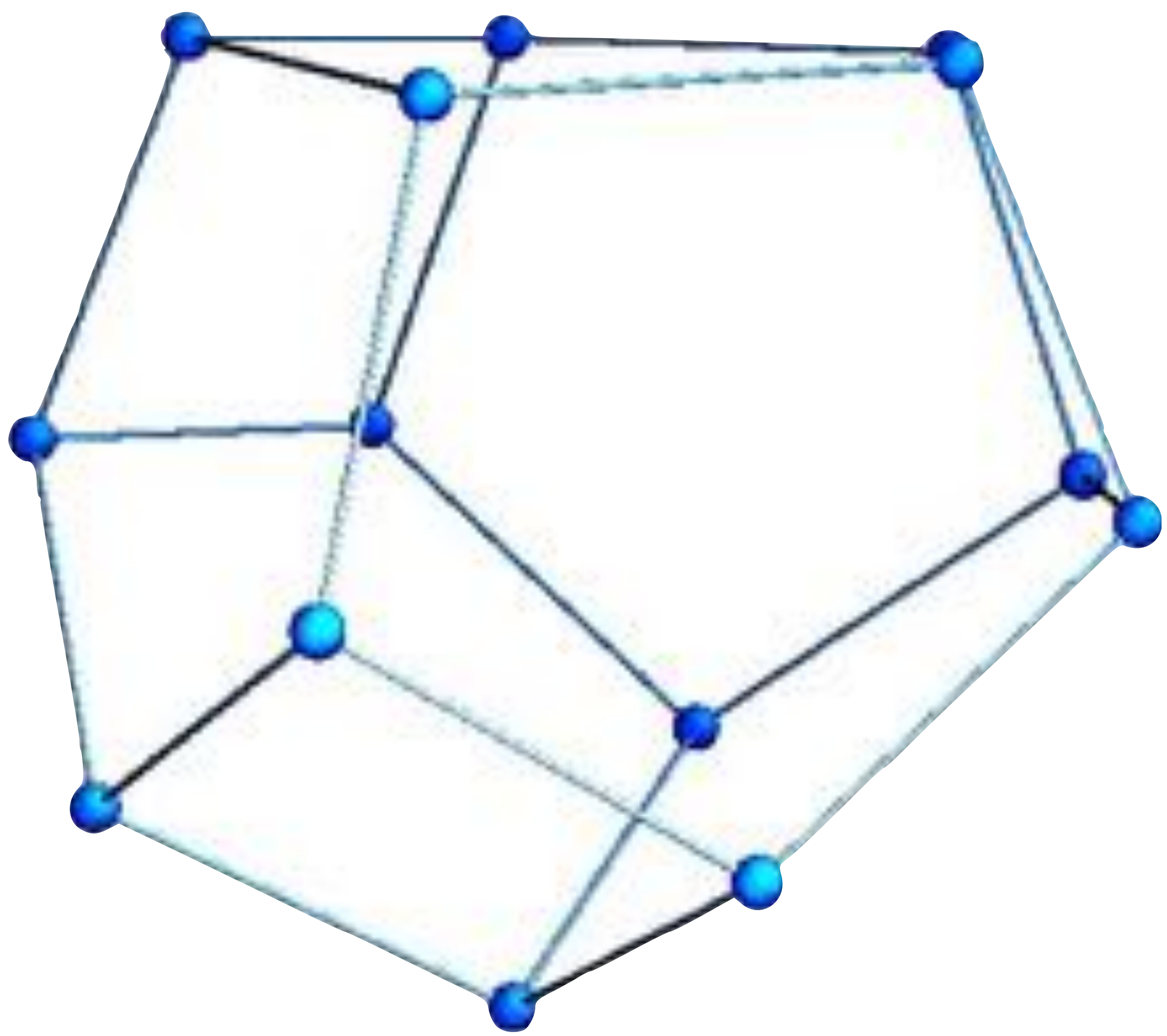




(generalized gauge freedom)

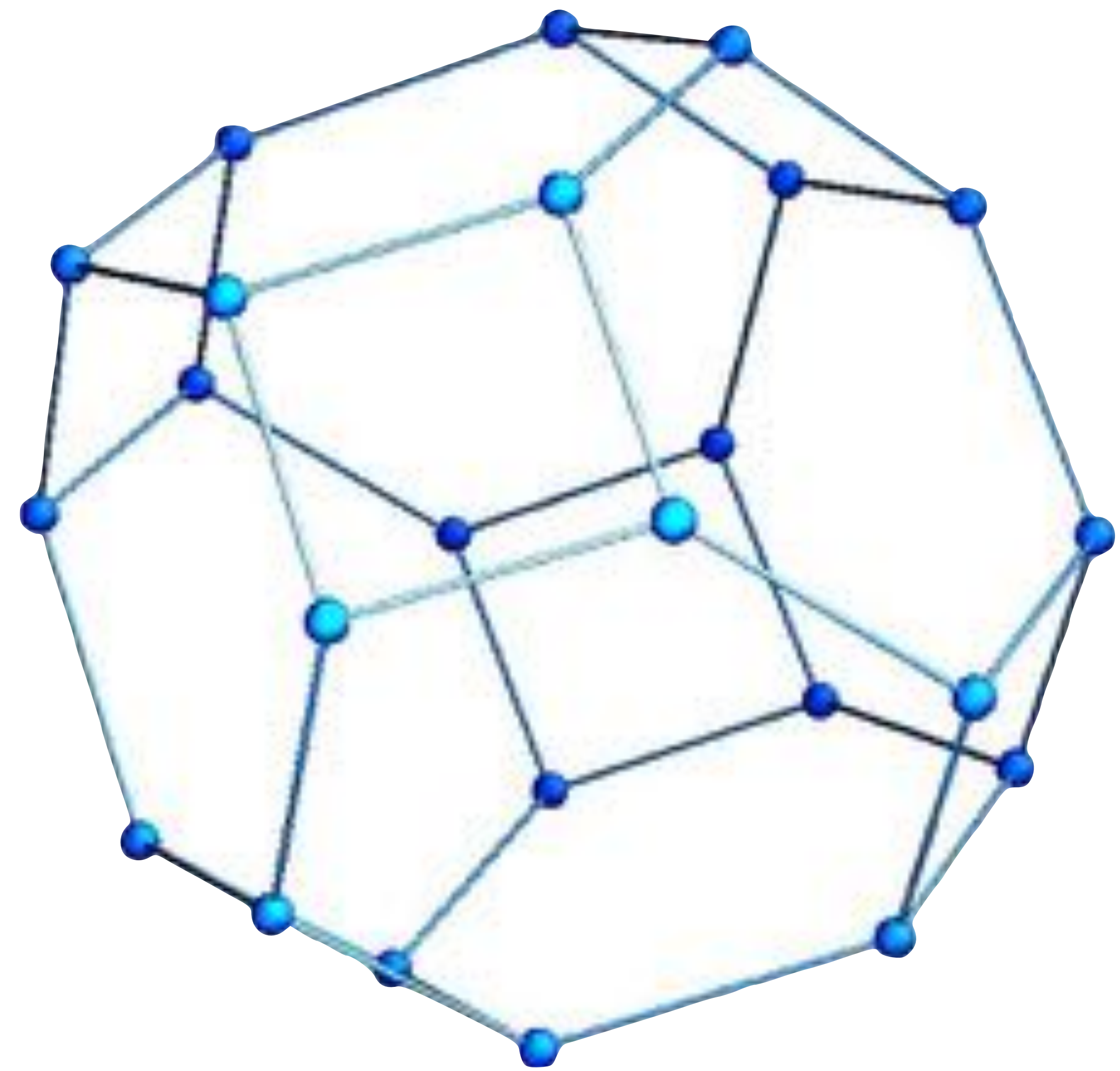
Building blocks at 6-points:

color-ordered amplitude



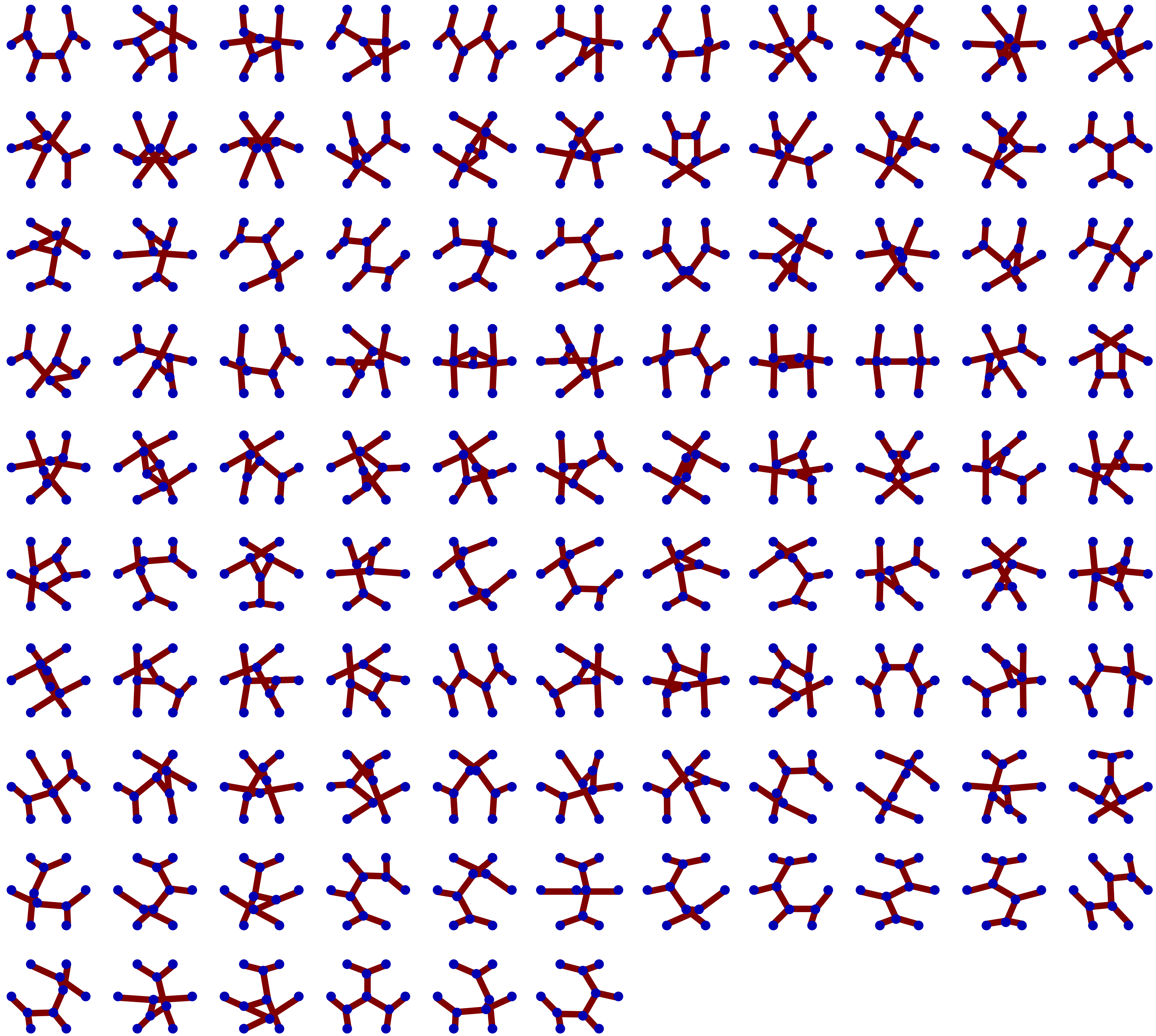
associahedron

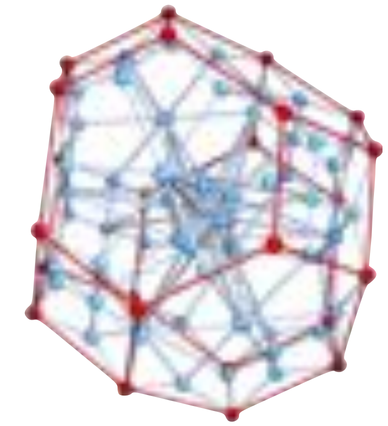
set of masters



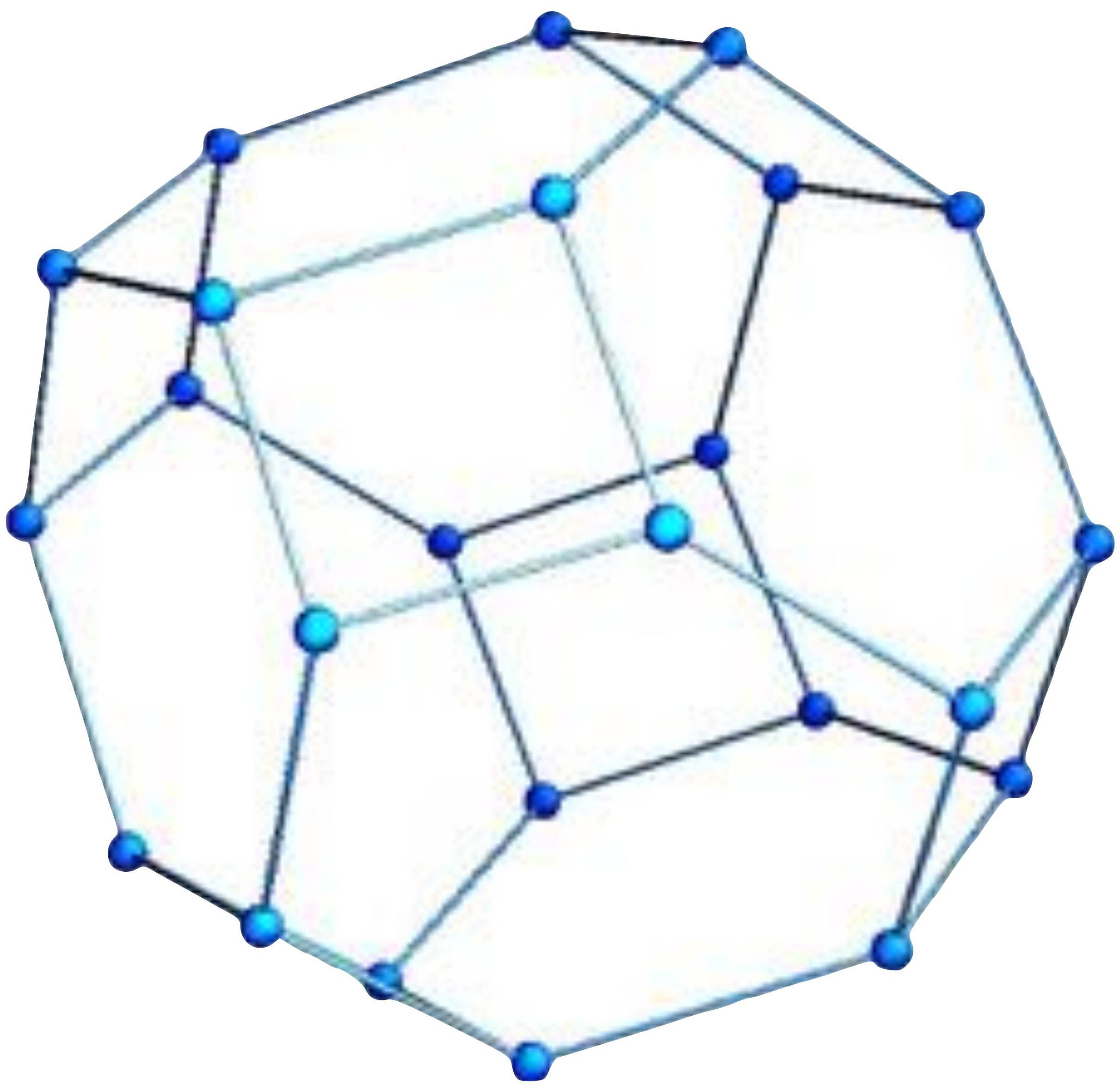
permutahedron

105
cubic graphs at 6 pt

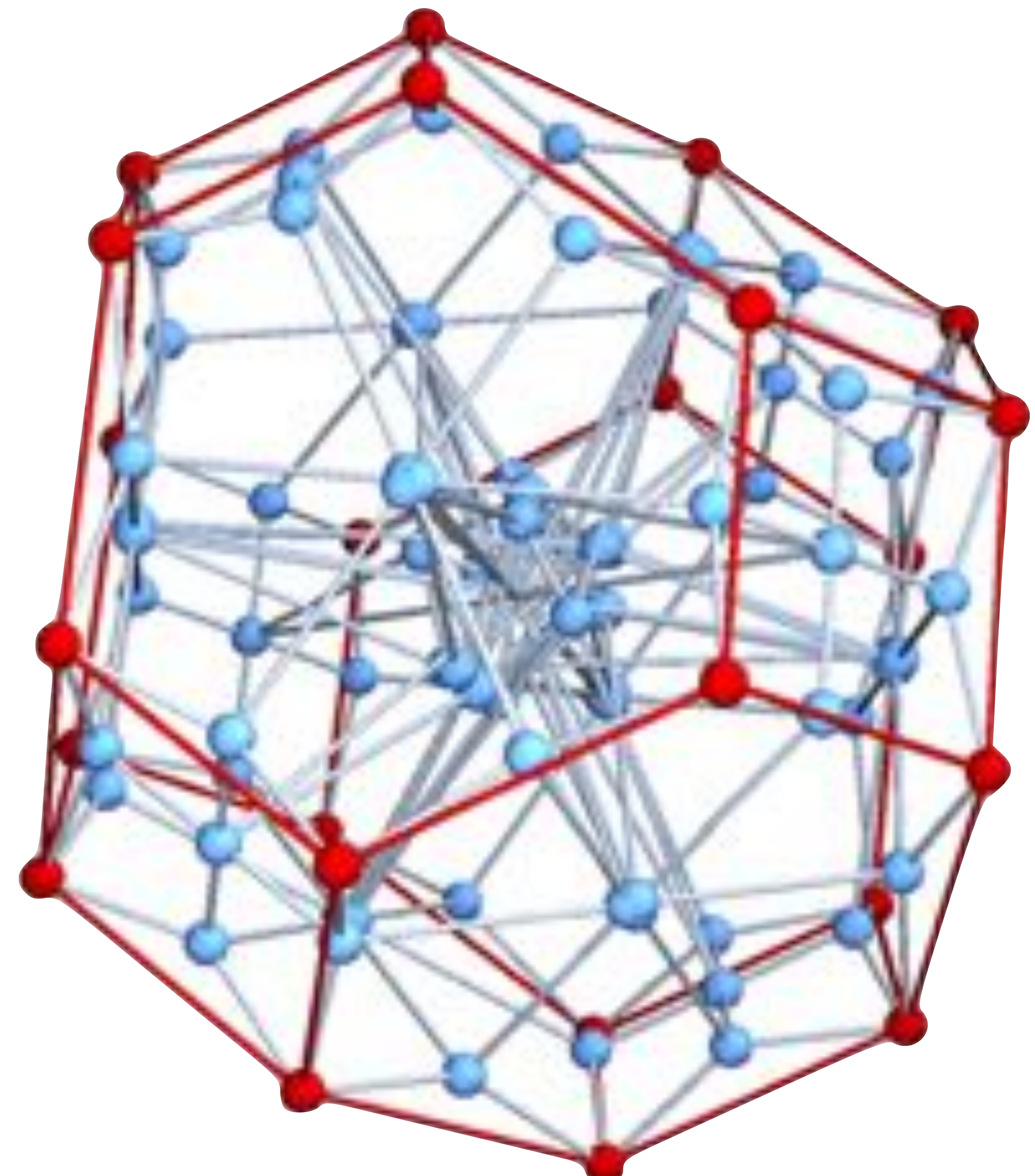


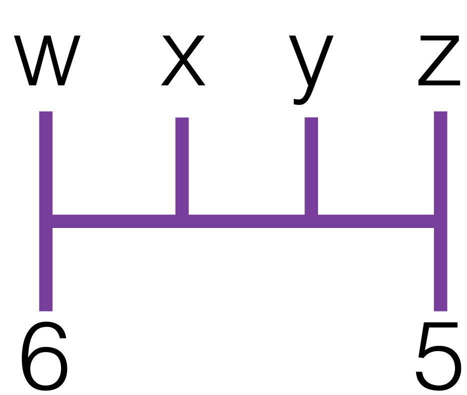


set of masters

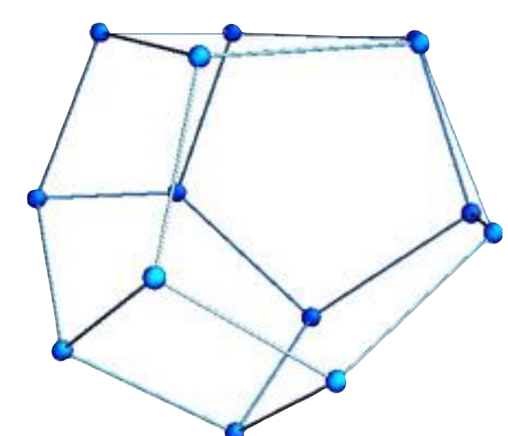


full amplitude



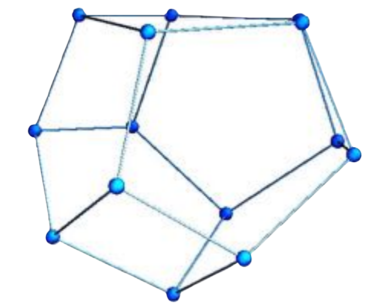


masters fixed by 6

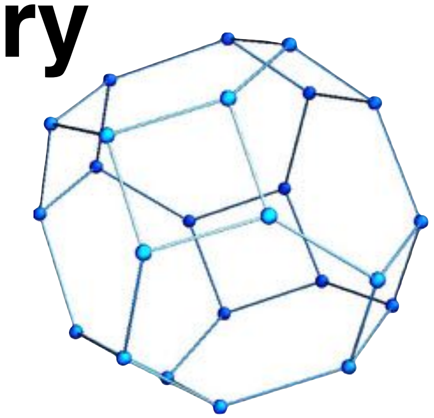


TREE-LEVEL SUMMARY

1. **Gauge invariant building blocks that speak to the theory:** color-ordered amplitudes, *associahedra*

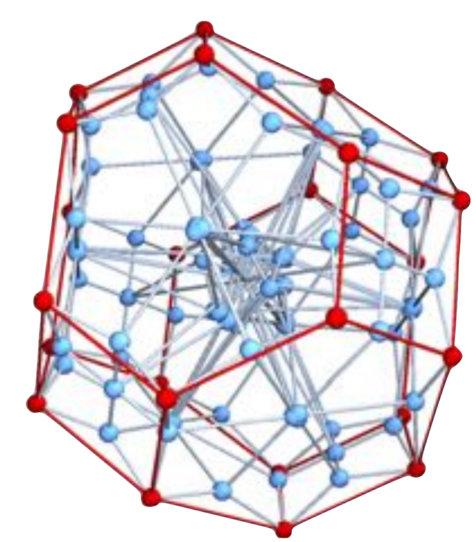


2. **CK means only need to specify the boundary data**: the master graphs, given by the relevant *permutahedron*



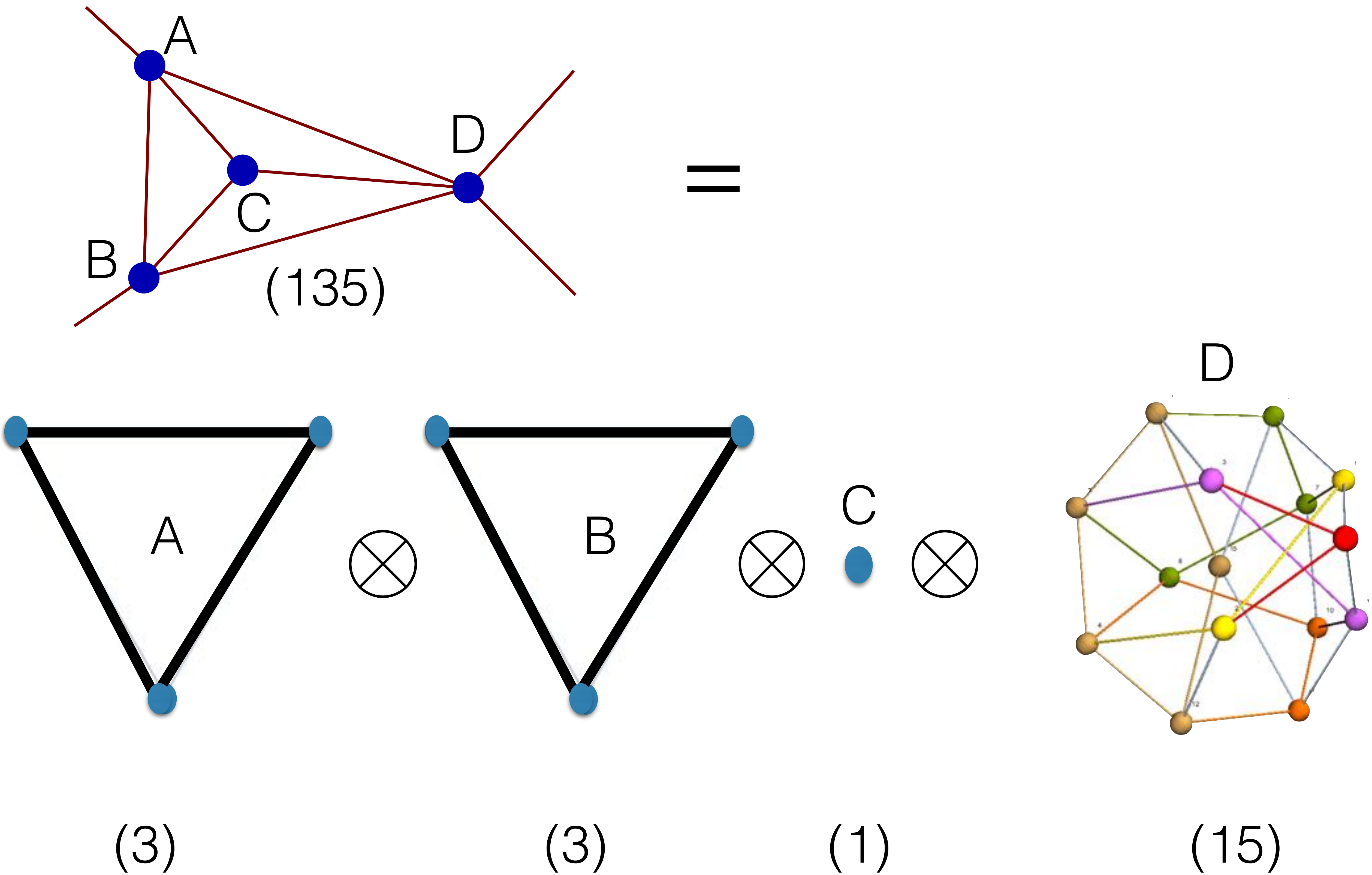
3. **Can solve for the *full amplitude efficiently* in terms of the (n-3)! independent *associohedra***

Hints that efficiency <—> geometry



4. Warmup: Color-Kinematics on cuts

Contributions to Color-Dressed Cut



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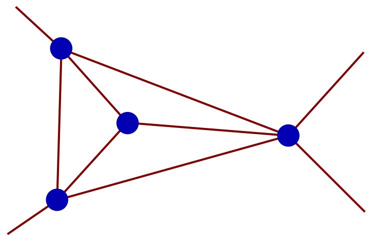
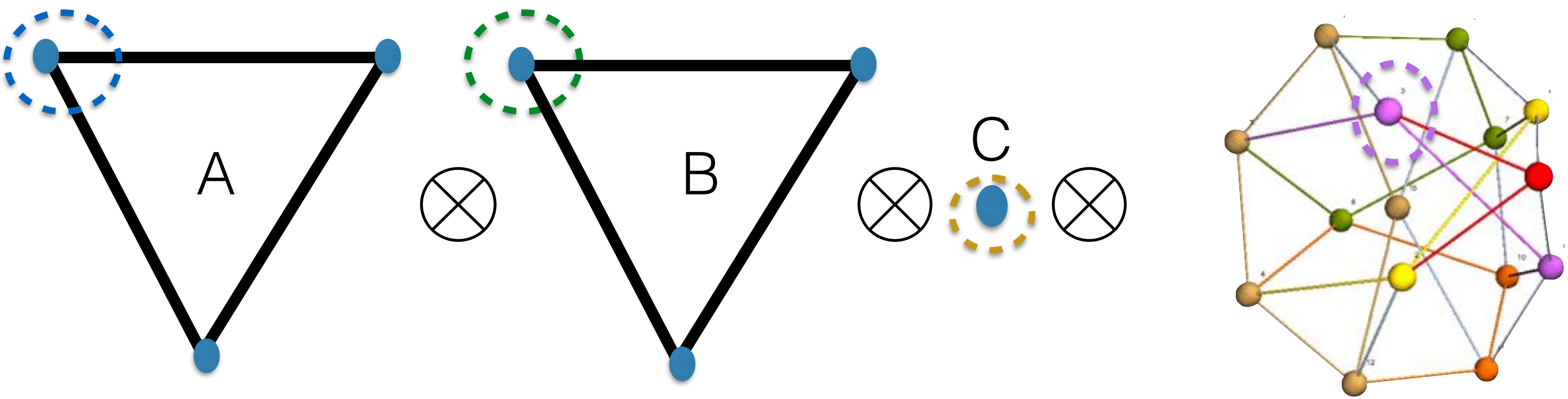
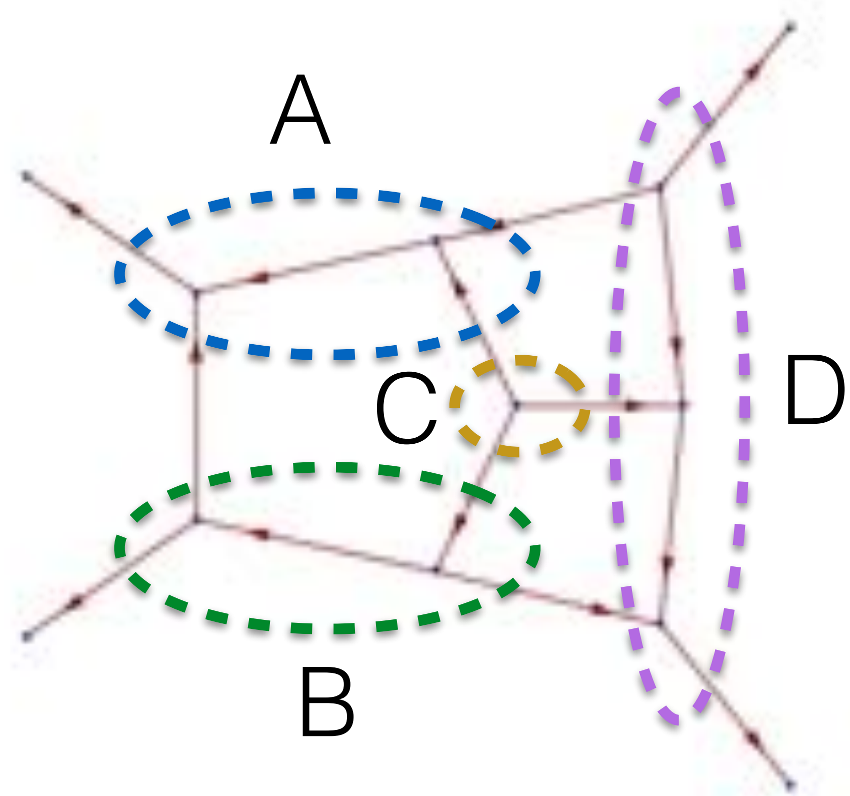




(3) (3) (1) (15)



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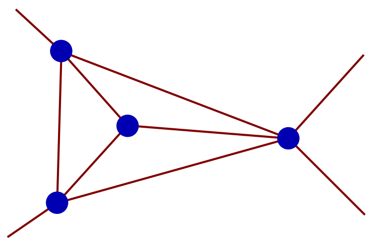

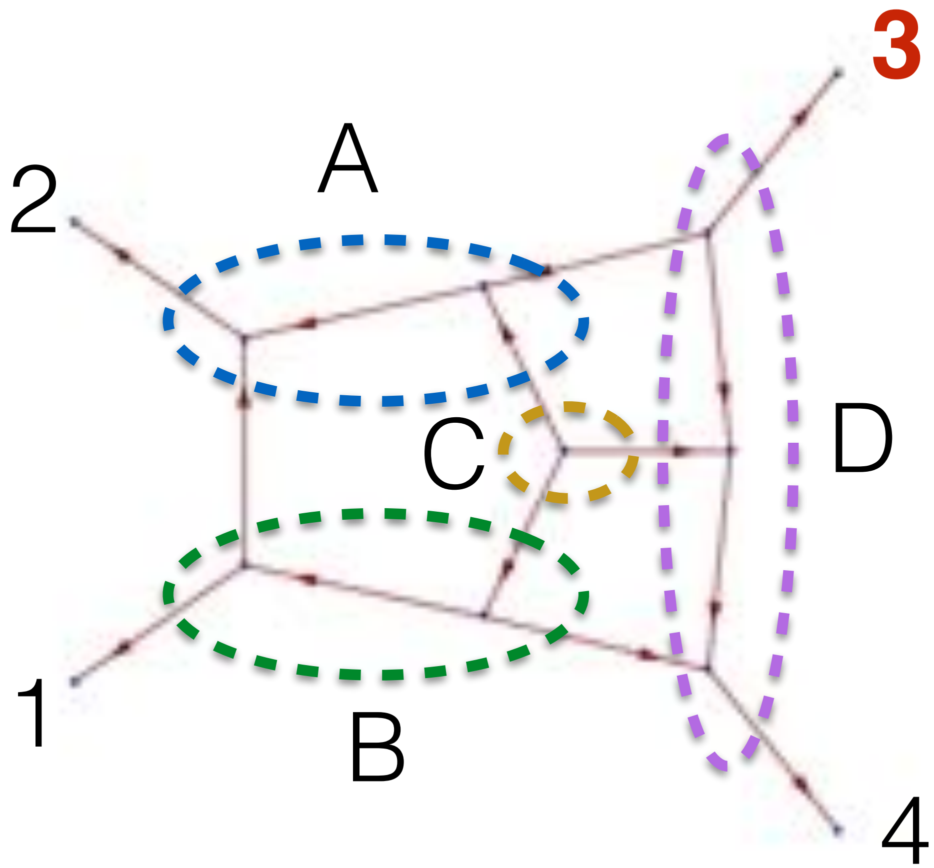

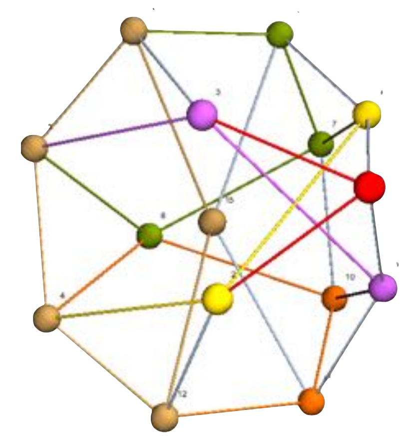

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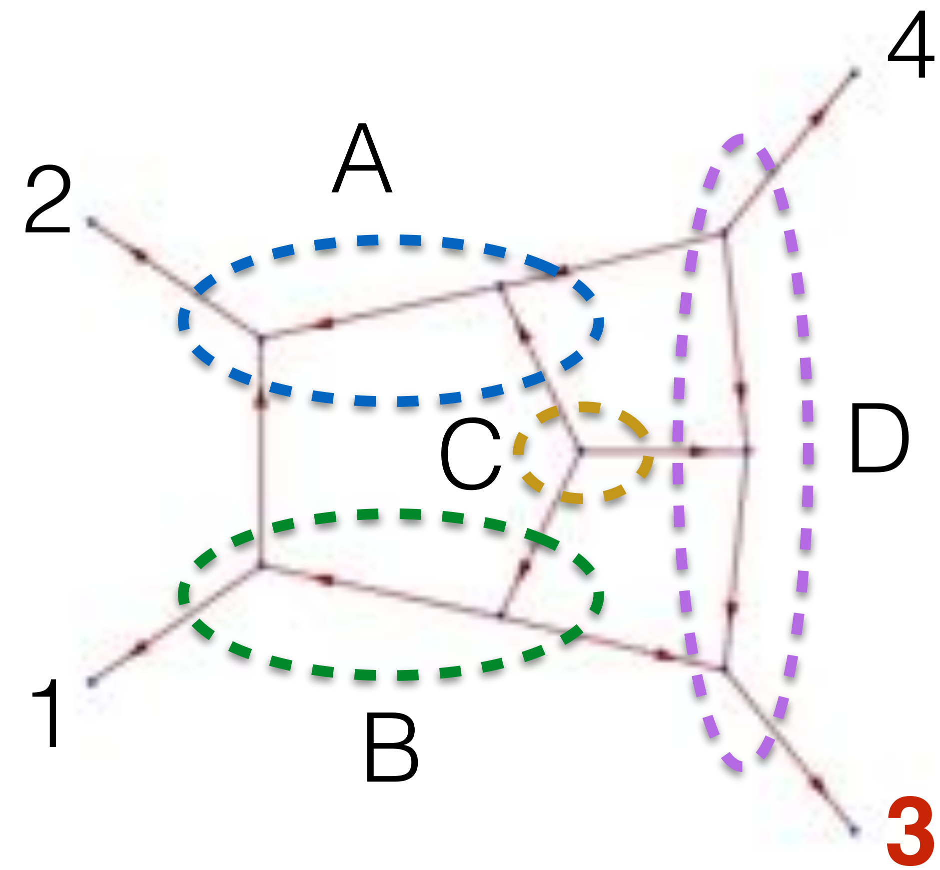
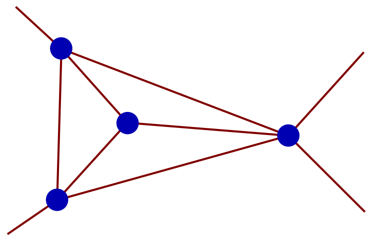


These are separate cut graphs in our graph of graphs.

Can identify linear Jacobis from acting $\hat{t}$ and $\hat{u}$ on every uncut edge — building every triangle — until closure.

Contributions to Color-Dressed Cut



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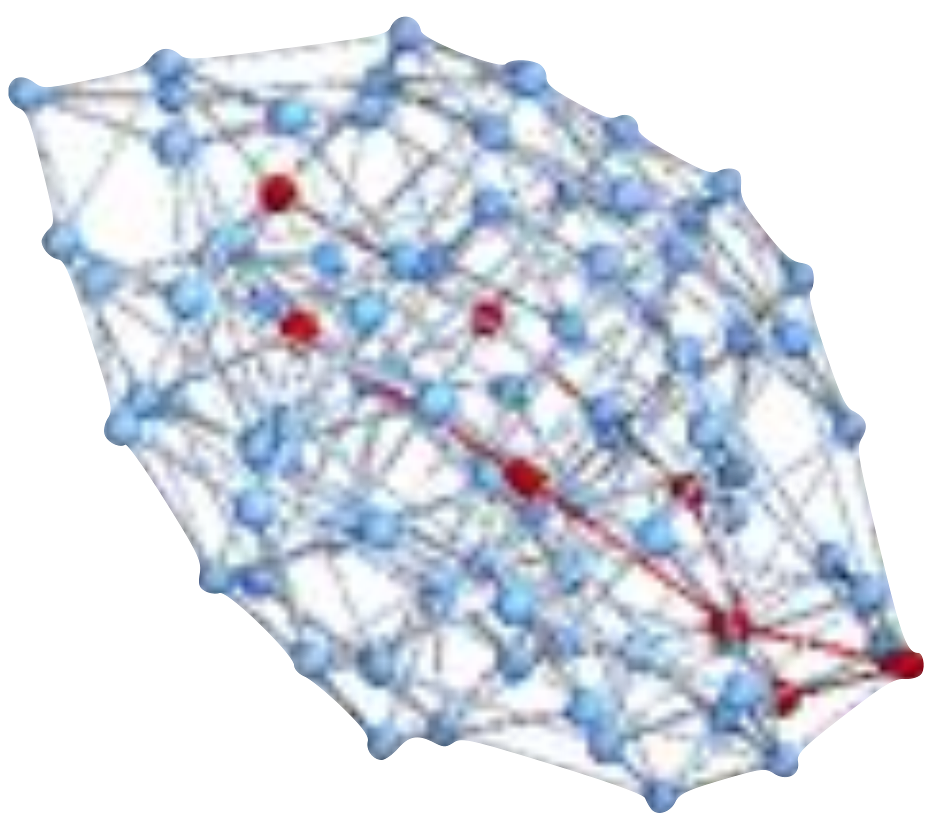
Drop unphysical graphs for theory.

92 cut-graphs contribute to this N=4 CD cut



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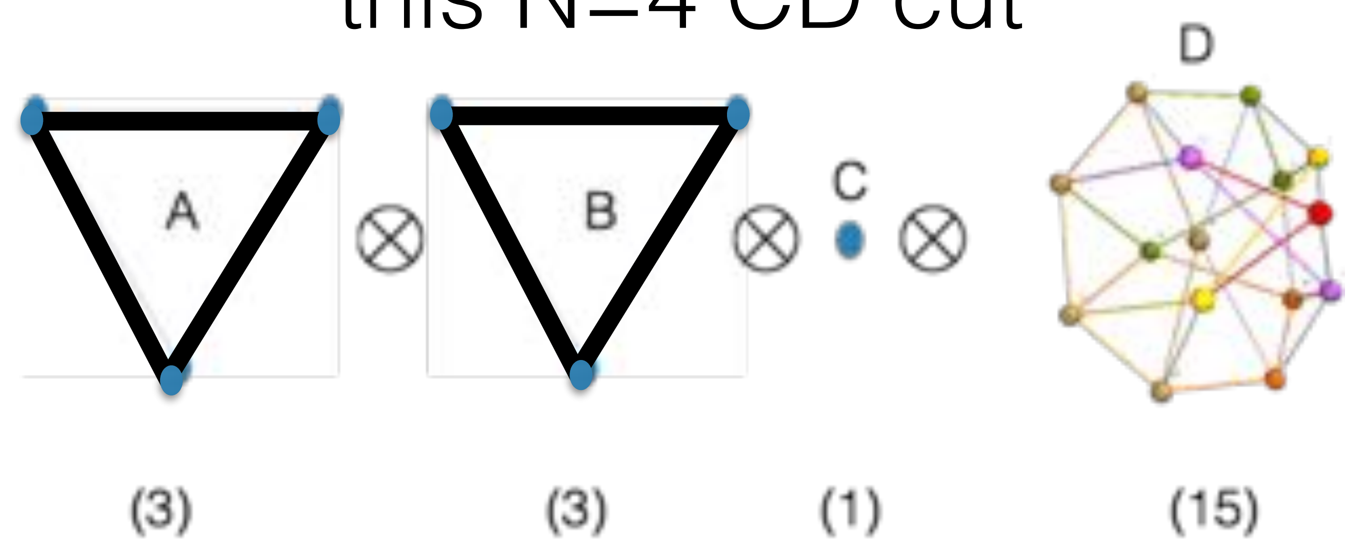
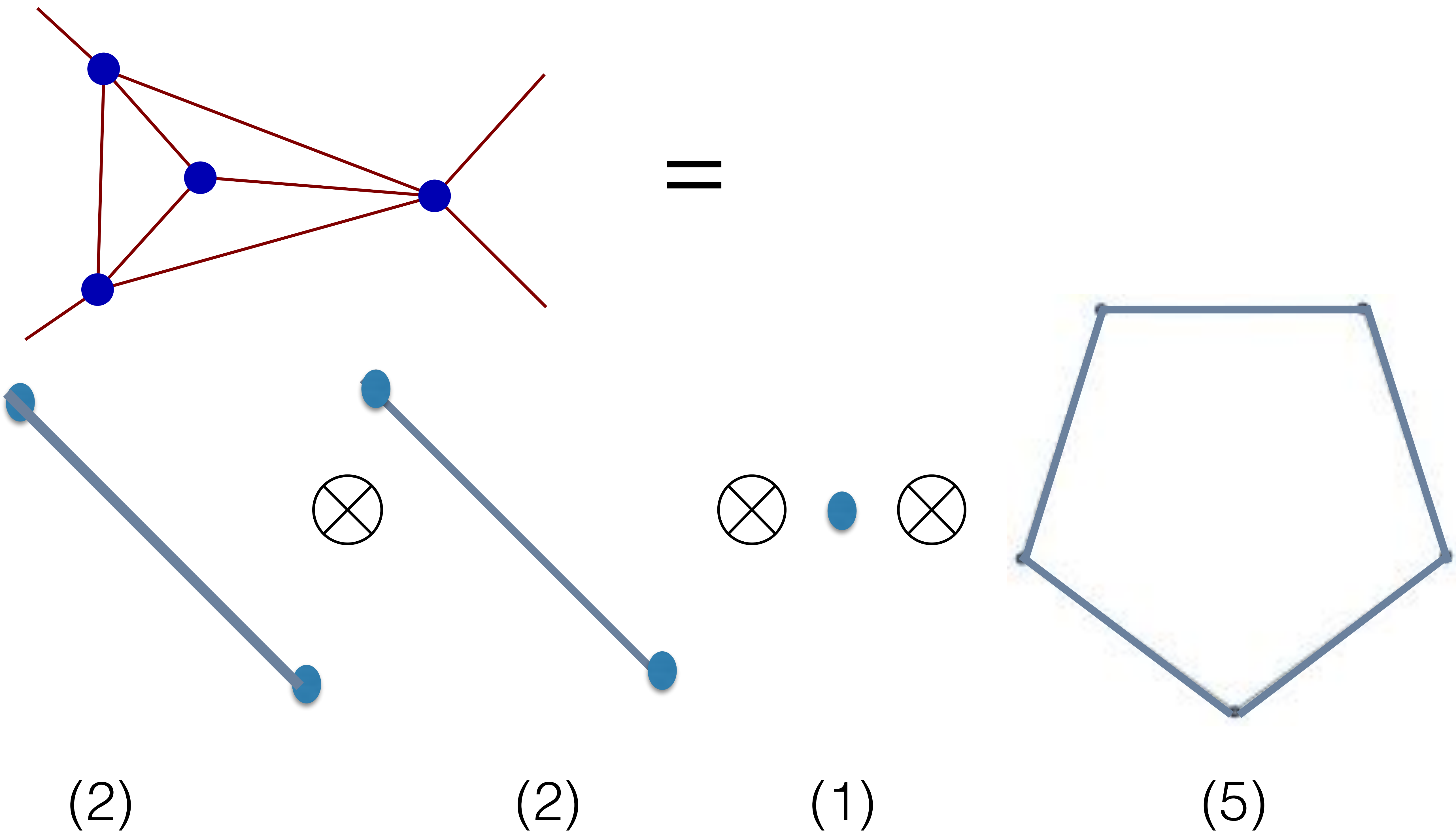


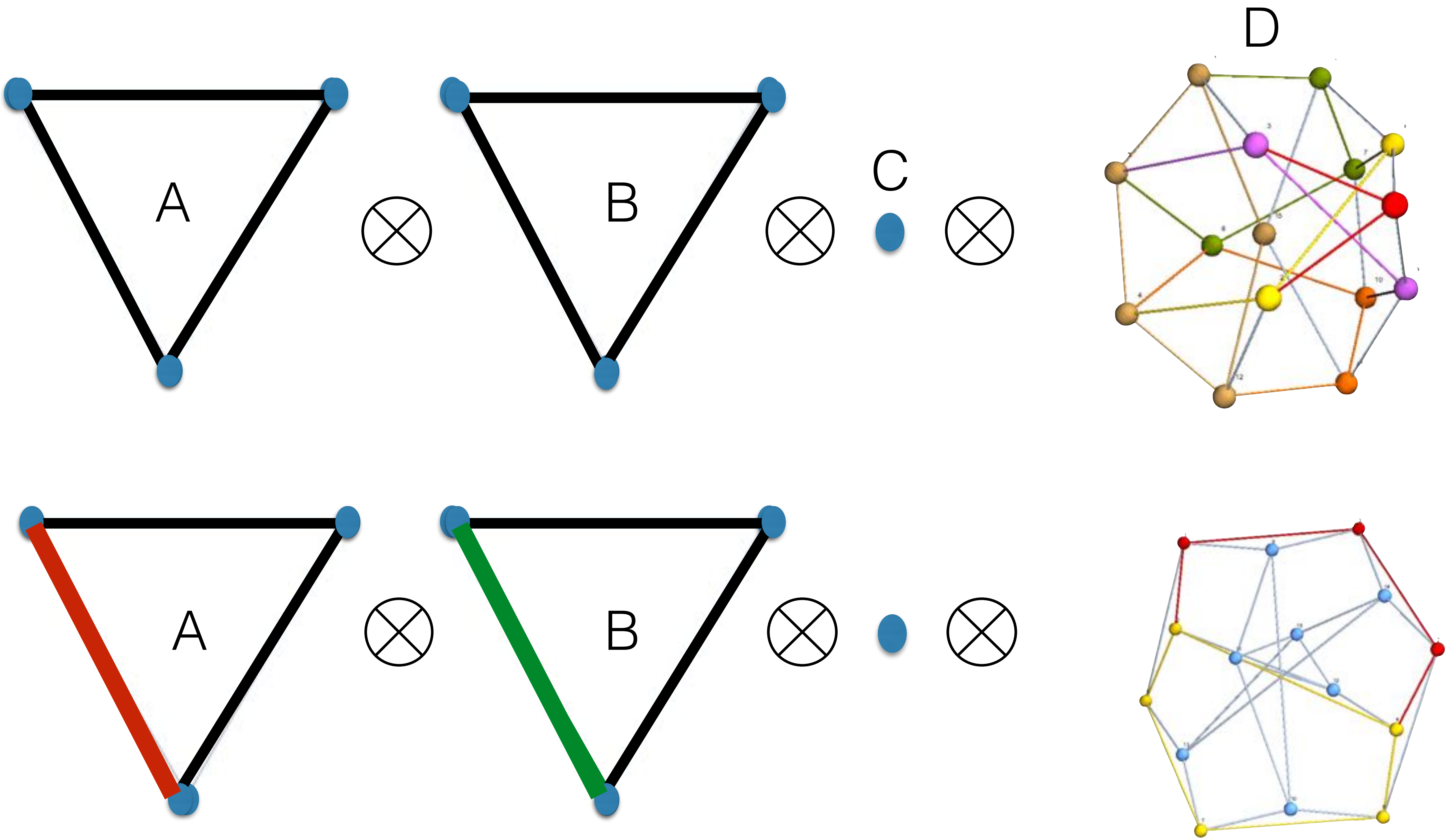
All cut-graphs specified by 8 masters

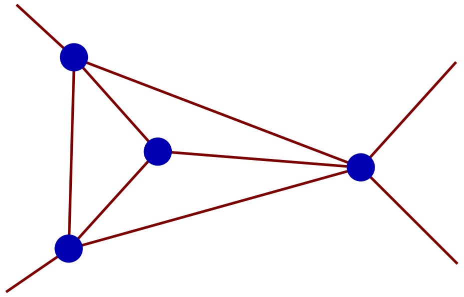
Only 2 of which need be non-vanishing: others pure gen. gauge-freedom.

Just like tree-level, can solve for master numerators in terms of color-ordered building blocks: color-ordered cuts



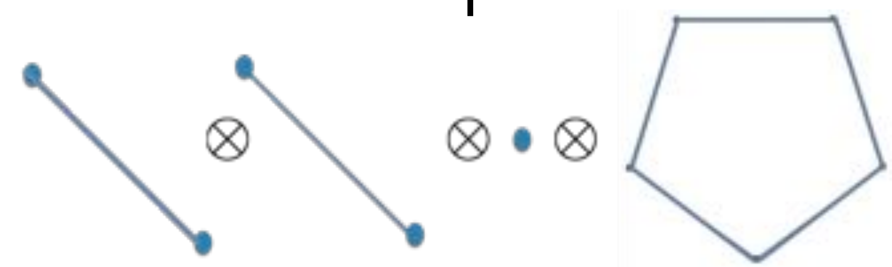
Just like tree-level, can solve for master numerators in terms of color-ordered building blocks: color-ordered cuts



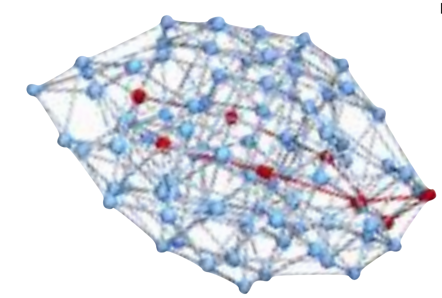


CUT SUMMARY

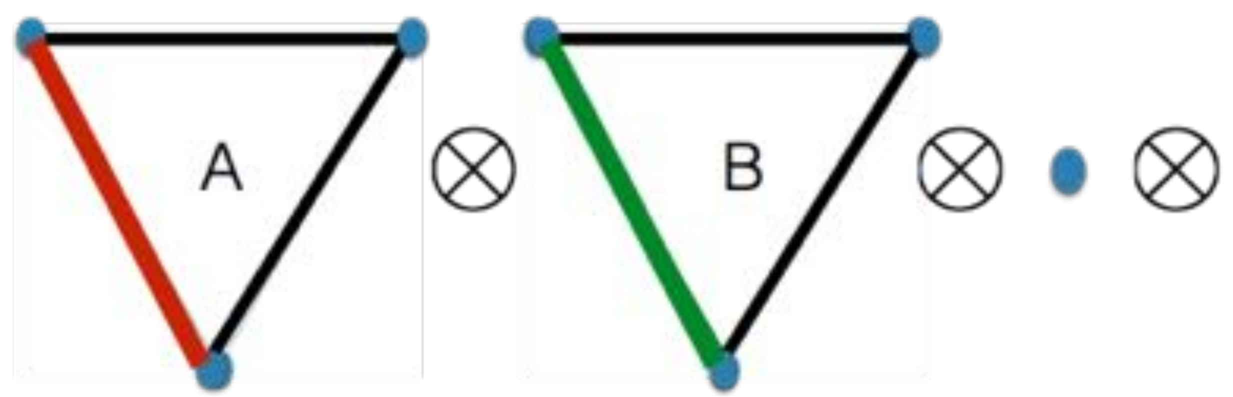
1. **Gauge invariant building blocks that speak to the theory:** color-ordered cuts — outer products of associahedra

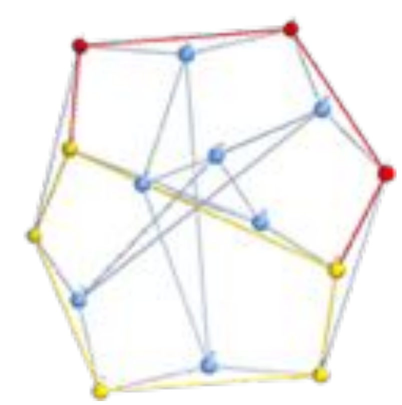


2. **CK means only needing to specify the boundary data**: n(g) of *contributing* master graphs



3. ***Can express these masters directly in terms of color-ordered cuts***



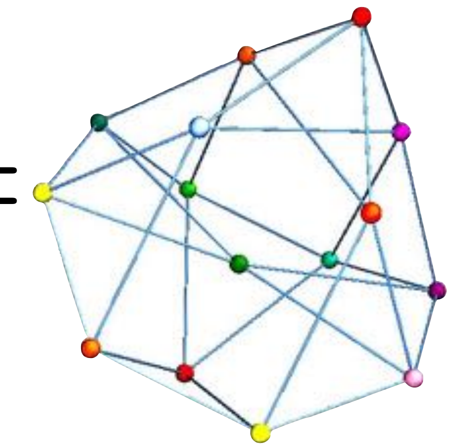


5. Off-shell pre-Integrand

Relax isomorphism, but achieve
Mulitloop color-kinematics without an ansatz

An algebraic loop-level approach

JJMC

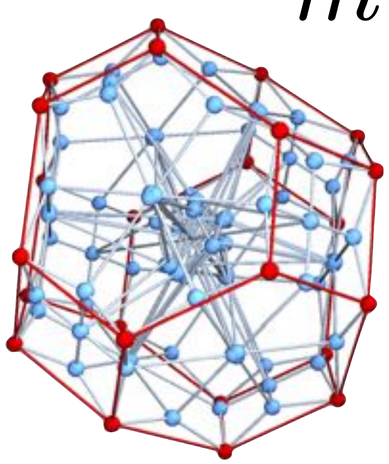
- Introduce multi-loop objects: pre-Integrands $\mathcal{I}_m^L =$ 
  - will contain all cut information manifestly, not functionally!
  - can decompose into color-stripped polytopes just like at tree-level or on cuts
  - introduce enough graphs to cover all labelings
  - each graph appears with fixed labels so can solve Jacobi's linearly

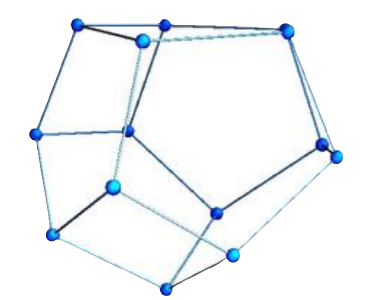
$$\{n_a + n_b + n_c = 0\} \rightarrow n_j = J_{jk} m_k$$

Asymmetric graphs contributing to Pre-Integrand: $\mathcal{I}_m^L$

- take all (2L +m)-point tree graphs
- identify 2L ext legs with +/- indep. off-shell loop momentum labels $\{l_1, -l_1, \ldots, -l_L, l_L\}$

$$\mathcal{I}_m^L = \sum_{j \in \text{assym}} \frac{n_j c_j}{d_j} \qquad \mathrm{I}_i = \sum_{j \in \text{perm}_i} \frac{n_j}{d_j}$$

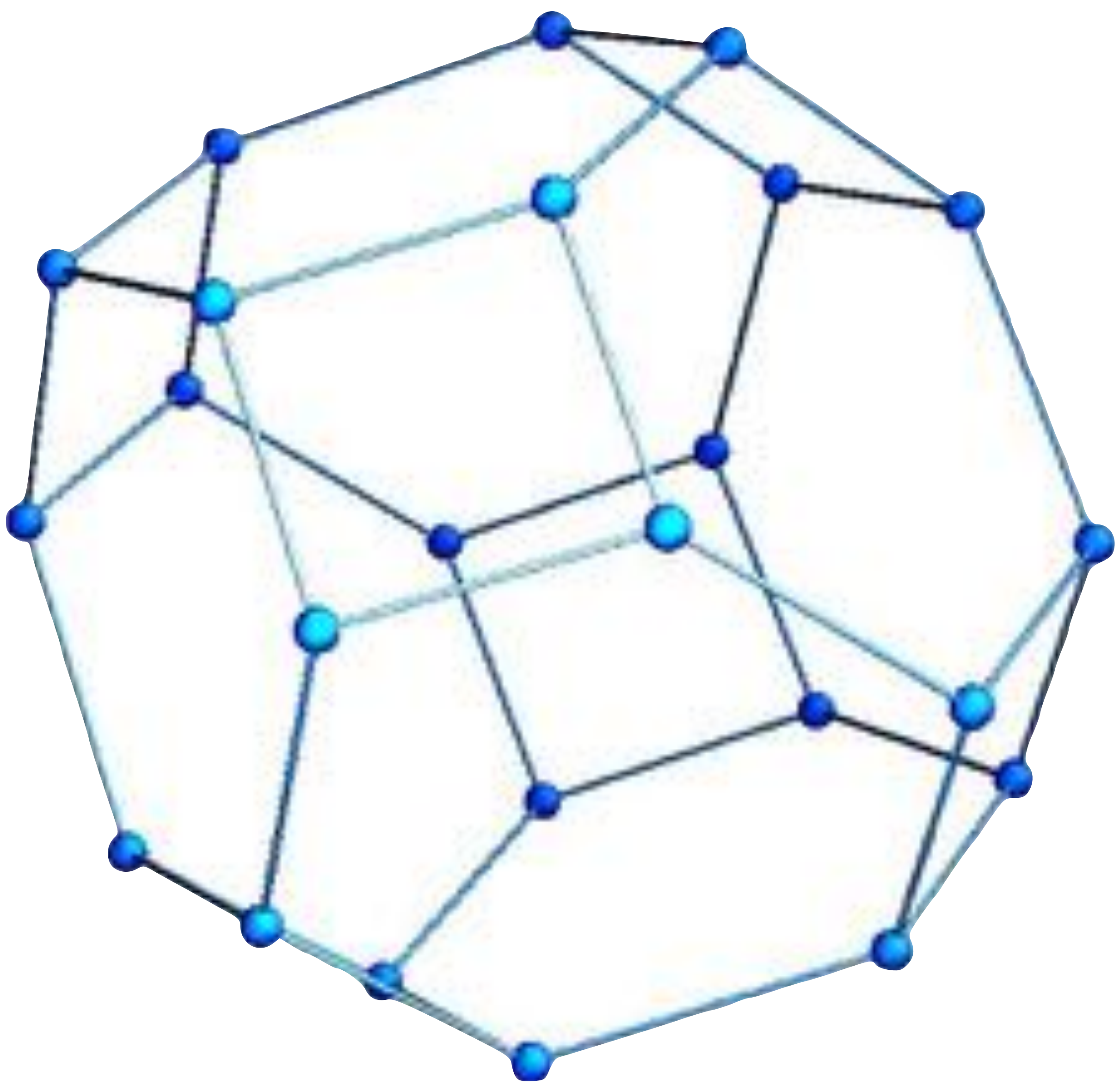




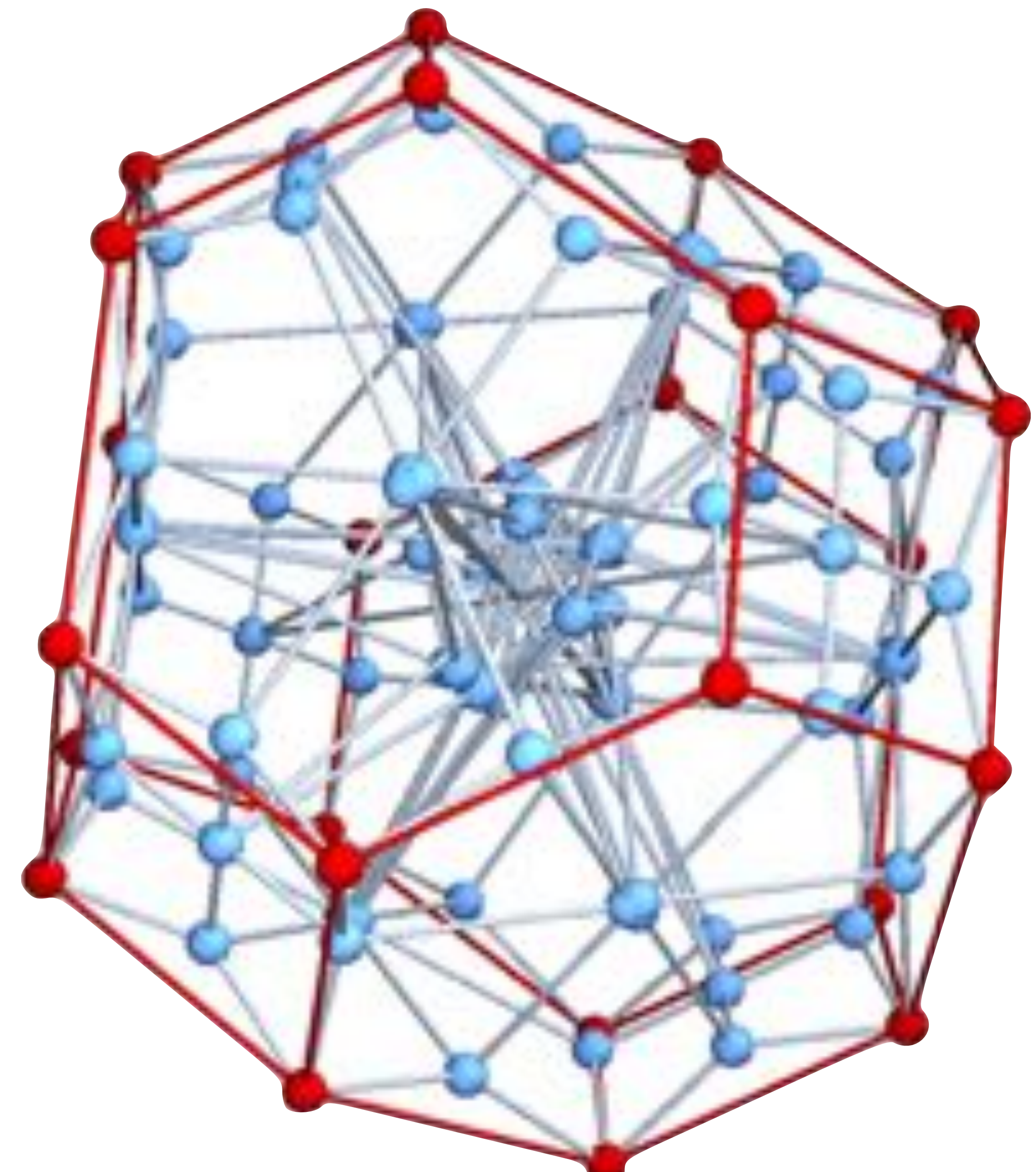
these labels dress all channels momentum can run through at L-loops M-point.

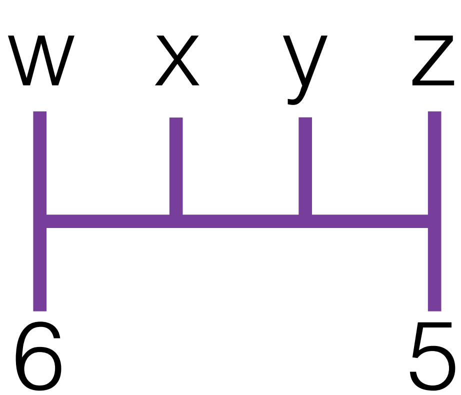
Recall 6-loop trees.

set of masters

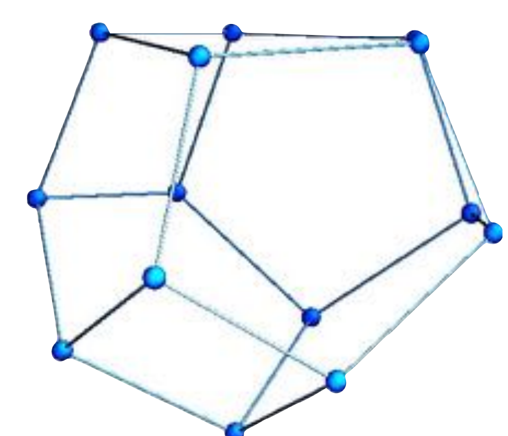


full tree amplitude



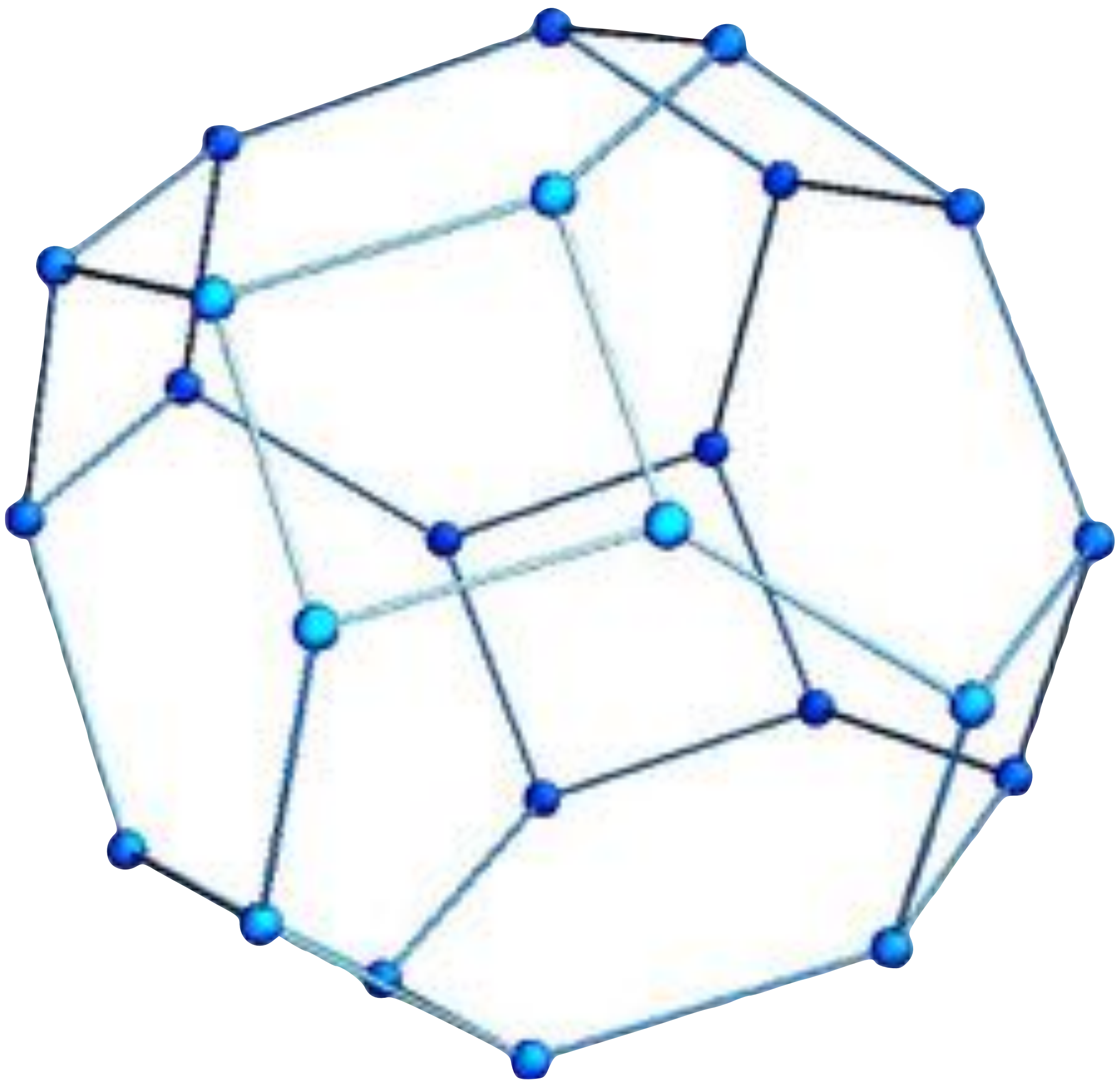


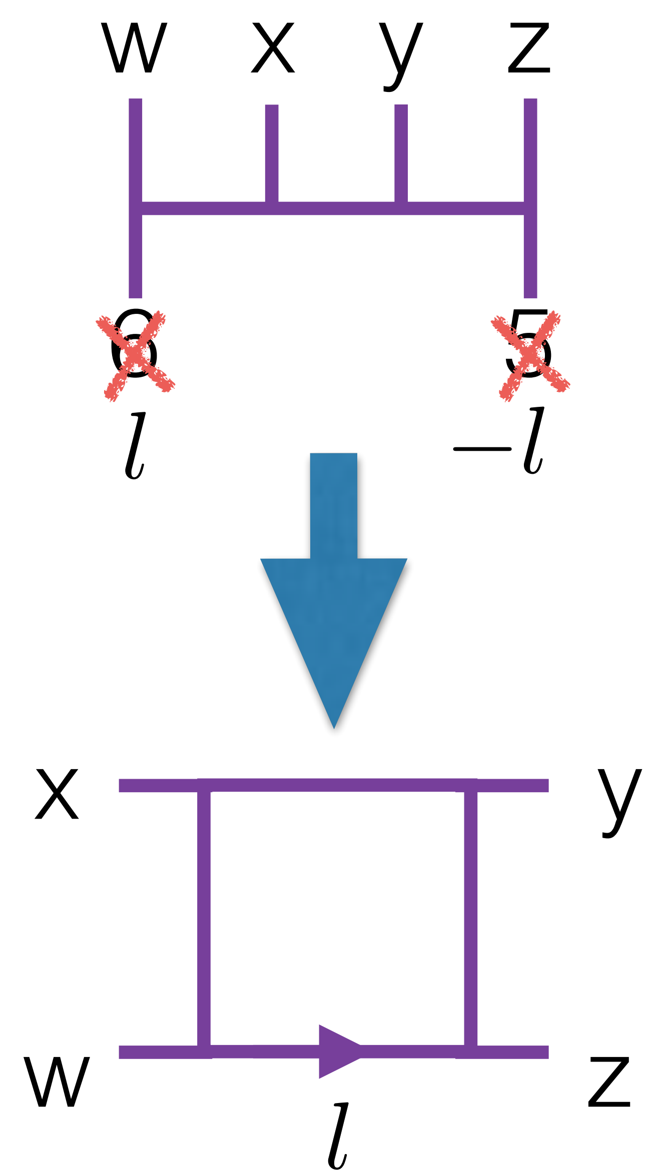
masters fixed by 6



Now we can talk interestingly about pre-Integrands of loops

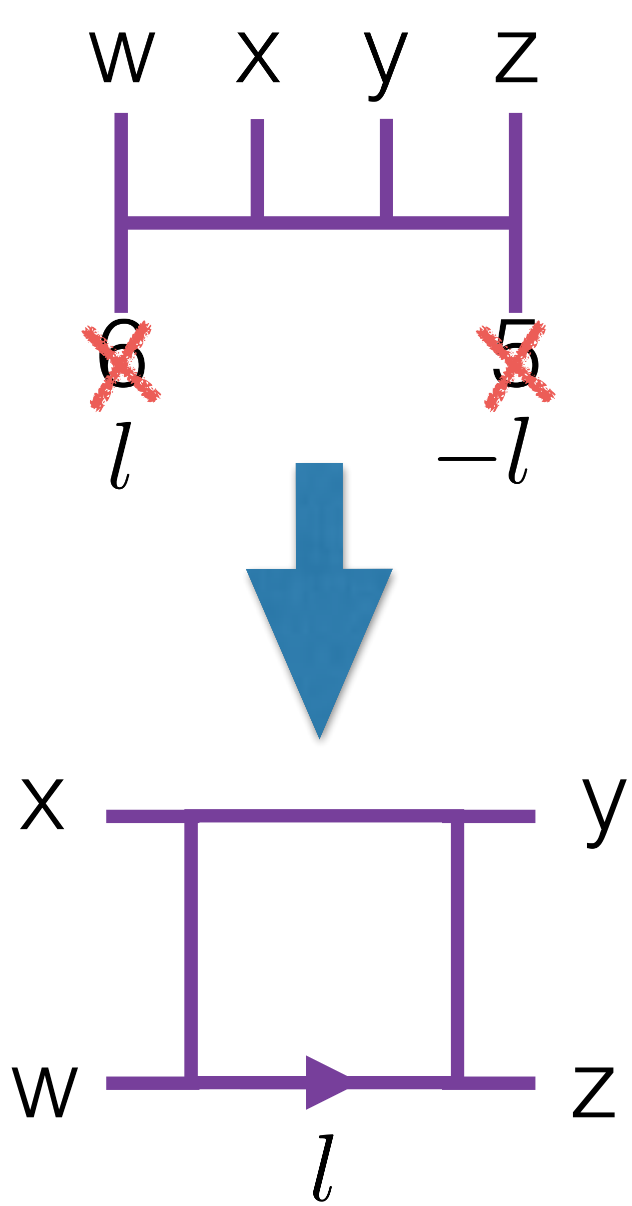
set of masters



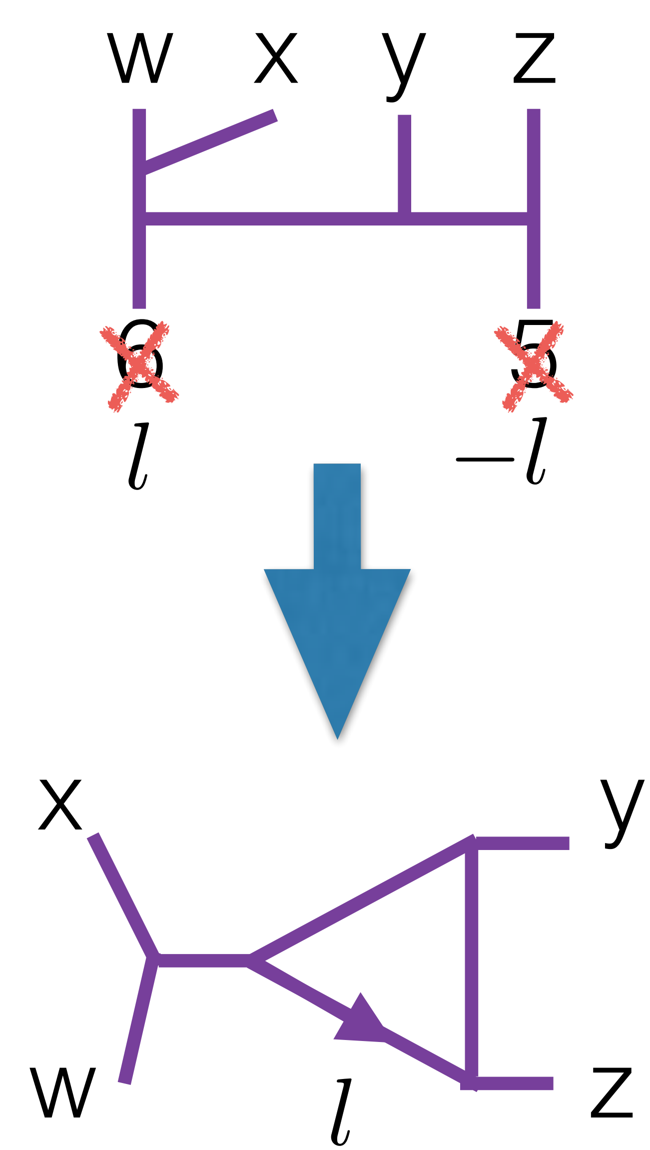


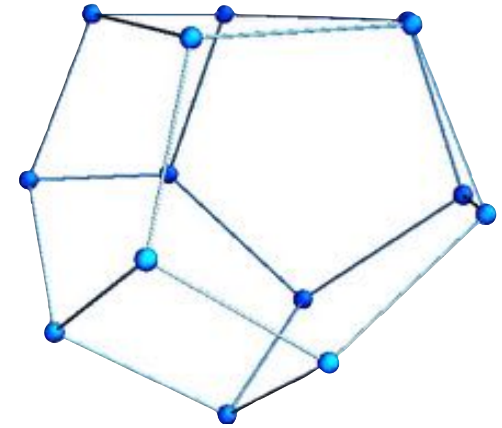
Now we can talk interestingly about pre-Integrands of loops

set of masters

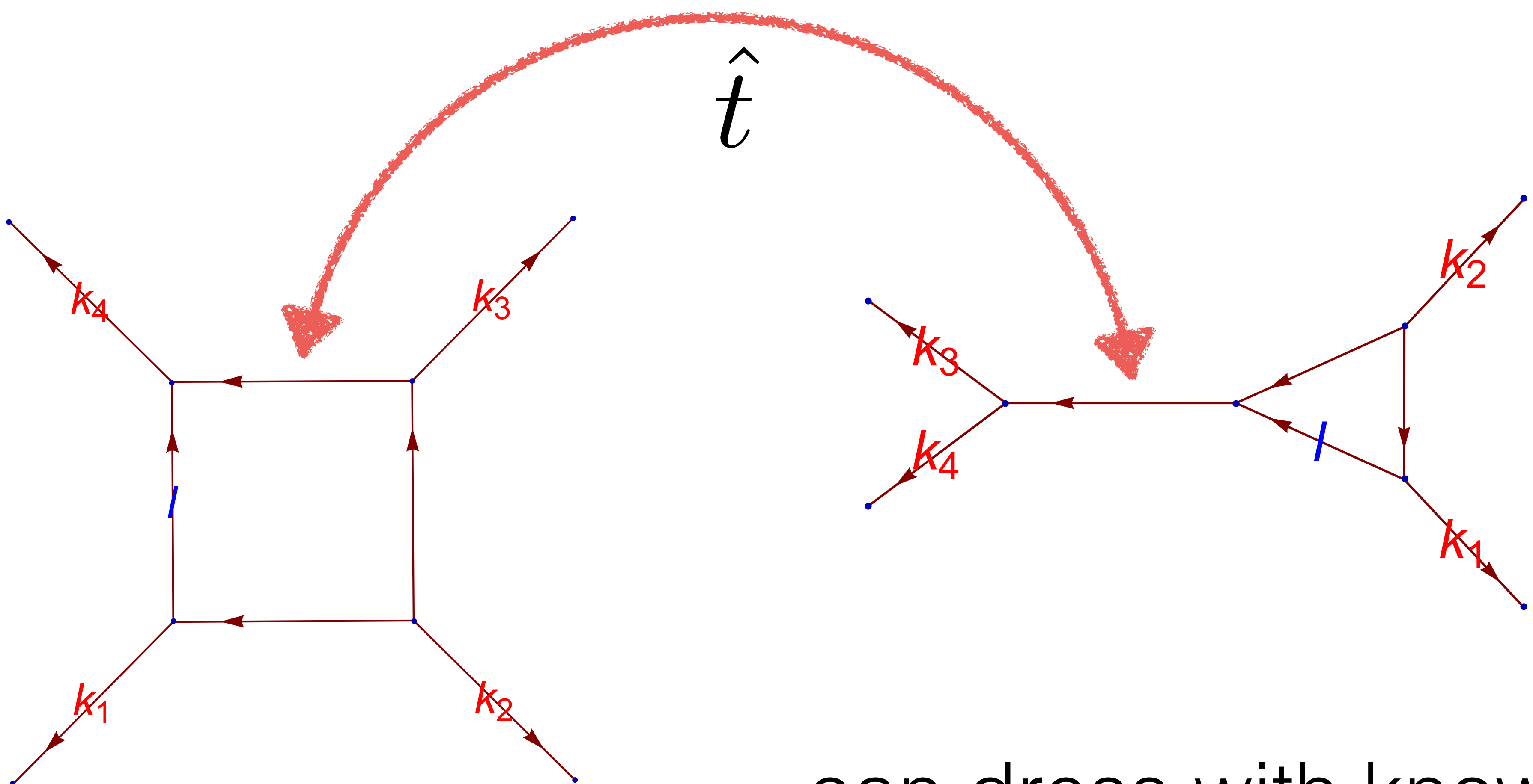


non-masters

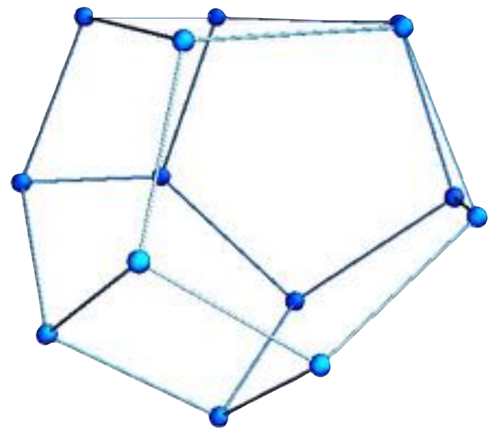


Any given building block 

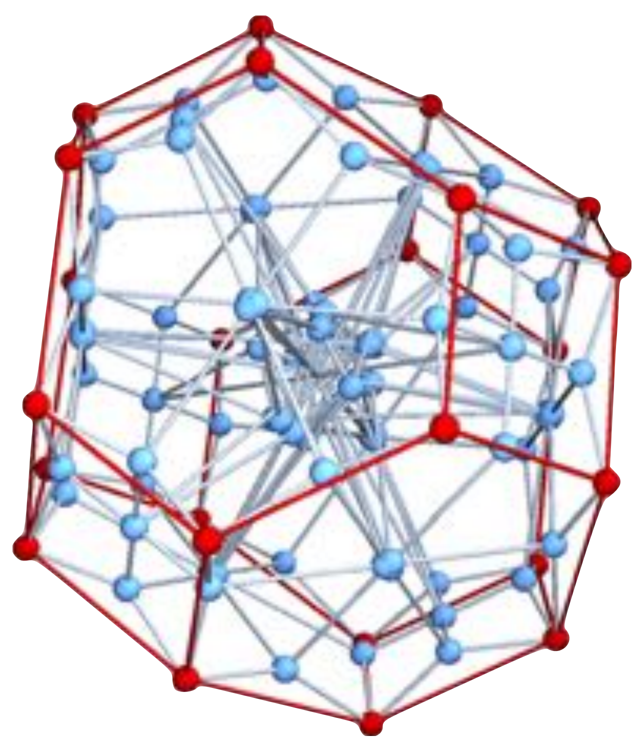
will be comprised of all the one-loop graphs labeled appropriately to the color-order:

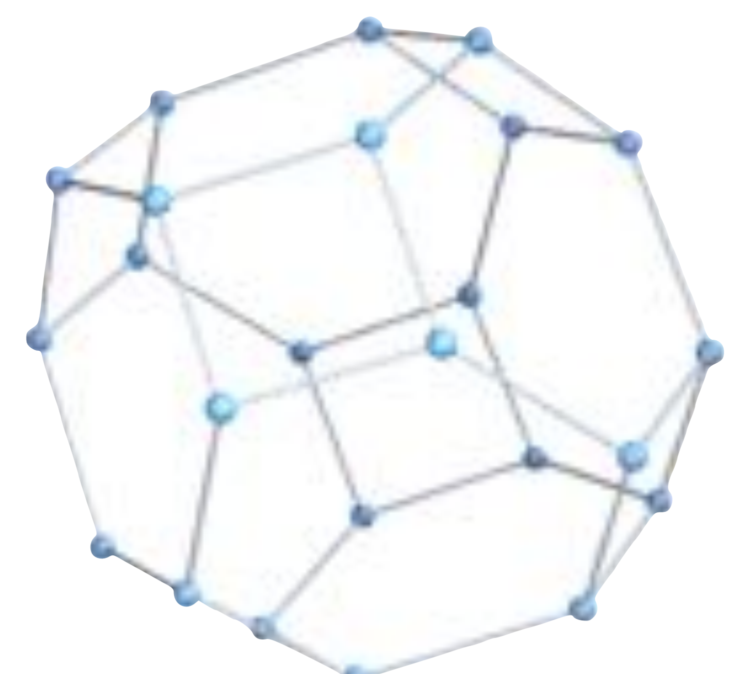


can dress with known off-shell information (unitarity, recursion, etc)

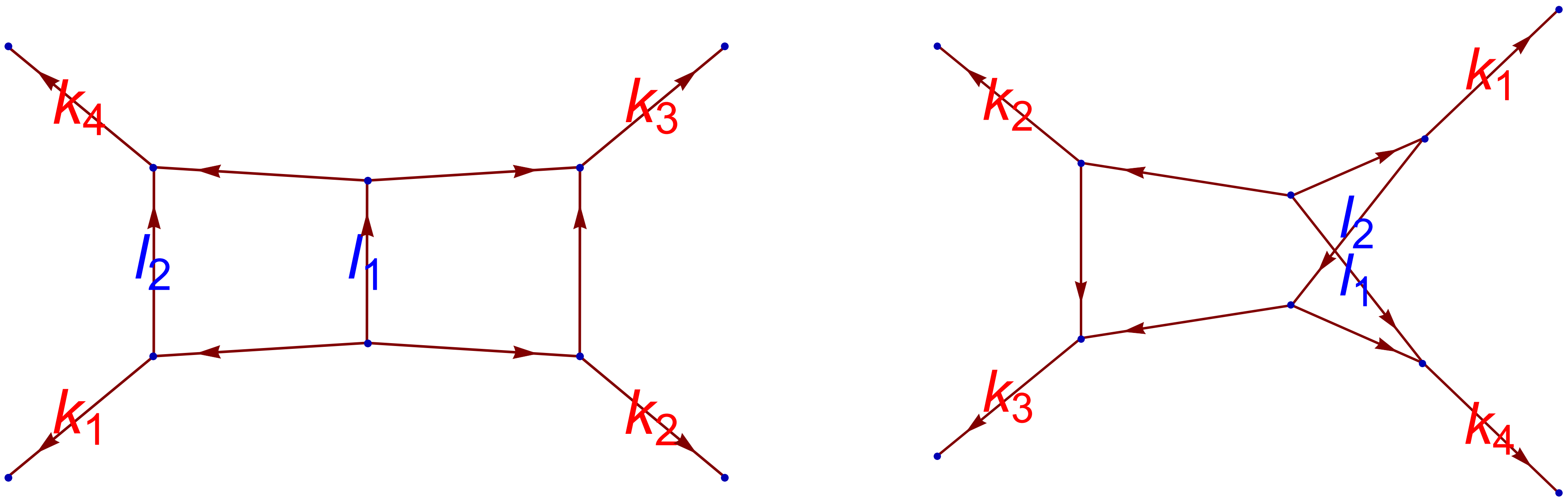
Any given  can dress with off-shell information (unitarity, recursion, etc)

does not need to come from a Jacobi satisfying representation. This will be boundary data. It just has to be true and off-shell on internal legs.

Then demand  satisfies Jacobi for a new rep.

and solve for new: 

For N=4 SYM at 4pt two-loop only need planar and non-planar boxes



+ perms

Jacobi eqns reduce all numerators to linear combination of two functions (2 asymmetric master graphs):

$$s\,(s\,t\,A(1234)) \qquad t\,(s\,t\,A(1234))$$

N=1 1-loop 4pt example:

7 off-shell masters, consider 7 color-ordered pre-Integrands

1st. in terms of relevant graphs

$$\mathbb{I}[\{-\ell, k_1, k_2, k_3, k_4, \ell\}] == \frac{n_1}{d_1} + \frac{n_{25}}{d_{25}} + \frac{n_{26}}{d_{26}} + \frac{n_{27}}{d_{27}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_3, \ell, k_4\}] == -\frac{n_1}{d_1} - \frac{n_2}{d_2} - \frac{n_5}{d_5} - \frac{n_{19}}{d_{19}} - \frac{n_{25}}{d_{25}} - \frac{n_{26}}{d_{26}} - \frac{n_{28}}{d_{28}} - \frac{n_{35}}{d_{35}} - \frac{n_{55}}{d_{55}} - \frac{n_{56}}{d_{56}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_4, k_3, \ell\}] == \frac{n_2}{d_2} - \frac{n_{27}}{d_{27}} + \frac{n_{28}}{d_{28}} + \frac{n_{29}}{d_{29}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_4, \ell, k_3\}] == -\frac{n_1}{d_1} - \frac{n_2}{d_2} - \frac{n_3}{d_3} - \frac{n_{13}}{d_{13}} - \frac{n_{25}}{d_{25}} - \frac{n_{28}}{d_{28}} - \frac{n_{29}}{d_{29}} - \frac{n_{31}}{d_{31}} - \frac{n_{47}}{d_{47}} - \frac{n_{48}}{d_{48}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, \ell, k_3, k_4\}] == \frac{n_2}{d_2} + \frac{n_5}{d_5} + \frac{n_6}{d_6} + \frac{n_{19}}{d_{19}} + \frac{n_{20}}{d_{20}} + \frac{n_{23}}{d_{23}} - \frac{n_{27}}{d_{27}} + \frac{n_{28}}{d_{28}} - \frac{n_{33}}{d_{33}} - \frac{n_{52}}{d_{52}} + \frac{n_{55}}{d_{55}} + \frac{n_{60}}{d_{60}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, \ell, k_4, k_3\}] == \frac{n_1}{d_1} + \frac{n_3}{d_3} + \frac{n_4}{d_4} + \frac{n_{13}}{d_{13}} + \frac{n_{14}}{d_{14}} + \frac{n_{17}}{d_{17}} + \frac{n_{25}}{d_{25}} + \frac{n_{27}}{d_{27}} + \frac{n_{33}}{d_{33}} + \frac{n_{47}}{d_{47}} + \frac{n_{52}}{d_{52}} + \frac{n_{53}}{d_{53}}$$

$$\mathbb{I}[\{-\ell, k_1, k_3, k_2, k_4, \ell\}] == \frac{n_3}{d_3} - \frac{n_{26}}{d_{26}} + \frac{n_{30}}{d_{30}} + \frac{n_{31}}{d_{31}}$$

N=1 1-loop 4pt example:

7 off-shell masters, consider 7 color-ordered pre-Integrands

2nd. in terms of 7 masters:

$$\mathbb{I}[\{-\ell, k_1, k_2, k_3, k_4, \ell\}] == \frac{n_4-n_6}{d_{27}} + \frac{n_5-n_6}{d_{26}} + \frac{n_3+n_5-n_6}{d_1} + \frac{n_3+n_5-n_6-n_7}{d_{25}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_3, \ell, k_4\}] == -\frac{n_5}{d_5} - \frac{n_3-n_4+n_5}{d_2} - \frac{n_5-n_6}{d_{26}} - \frac{n_5-n_6}{d_{35}} - \frac{n_5-n_6}{d_{56}} - \frac{n_3+n_5-n_6}{d_1} - \frac{n_3+n_5-n_6-n_7}{d_{25}} - \frac{n_3+n_5-n_6-n_7}{d_{28}} - \frac{n_3+n_5-n_6-n_7}{d_{55}} - \frac{n_5-n_9+n_{10}}{d_{19}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_4, k_3, \ell\}] == \frac{n_3-n_4}{d_{29}} + \frac{n_3-n_4+n_5}{d_2} - \frac{n_4-n_6}{d_{27}} + \frac{n_3+n_5-n_6-n_7}{d_{28}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_4, \ell, k_3\}] == -\frac{n_3}{d_3} - \frac{n_3-n_4}{d_{29}} - \frac{n_3-n_4}{d_{31}} - \frac{n_3-n_4}{d_{48}} - \frac{n_3-n_4+n_5}{d_2} - \frac{n_3+n_5-n_6}{d_1} - \frac{n_3+n_5-n_6-n_7}{d_{25}} - \frac{n_3+n_5-n_6-n_7}{d_{28}} - \frac{n_3+n_5-n_6-n_7}{d_{47}} - \frac{n_3-n_7+n_9}{d_{13}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, \ell, k_3, k_4\}] == \frac{n_5}{d_5} + \frac{n_3-n_4+n_5}{d_2} - \frac{n_4-n_6}{d_{27}} - \frac{n_4-n_6}{d_{33}} - \frac{n_4-n_6}{d_{52}} + \frac{n_6}{d_6} + \frac{n_3+n_5-n_6-n_7}{d_{28}} + \frac{n_3+n_5-n_6-n_7}{d_{55}} + \frac{n_3+n_5-n_6-n_7}{d_{60}} + \frac{n_6-n_7+n_{10}}{d_{23}} + \frac{n_5-n_9+n_{10}}{d_{19}} + \frac{n_6-n_9+n_{10}}{d_{20}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, \ell, k_4, k_3\}] == \frac{n_3}{d_3} + \frac{n_4}{d_4} + \frac{n_4-n_6}{d_{27}} + \frac{n_4-n_6}{d_{33}} + \frac{n_4-n_6}{d_{52}} + \frac{n_3+n_5-n_6}{d_1} + \frac{n_3+n_5-n_6-n_7}{d_{25}} + \frac{n_3+n_5-n_6-n_7}{d_{47}} + \frac{n_3+n_5-n_6-n_7}{d_{53}} + \frac{n_3-n_7+n_9}{d_{13}} + \frac{n_4-n_7+n_9}{d_{14}} + \frac{n_4-n_7+n_{10}}{d_{17}}$$

$$\mathbb{I}[\{-\ell, k_1, k_3, k_2, k_4, \ell\}] == \frac{n_3}{d_3} + \frac{n_3-n_4}{d_{31}} - \frac{n_5-n_6}{d_{26}} + \frac{n_7-n_9}{d_{30}}$$

CAN INVERT OFF-SHELL:

$$\ell^2 \neq 0$$

7 independent CO Pre-Intg

N=1 1-loop 4pt example:

7 off-shell masters, consider 7 color-ordered pre-Integrands

2nd. in terms of 7 masters:

$$\mathbb{I}[\{-\ell, k_1, k_2, k_3, k_4, \ell\}] == \frac{n_4-n_6}{d_{27}} + \frac{n_5-n_6}{d_{26}} + \frac{n_3+n_5-n_6}{d_1} + \frac{n_3+n_5-n_6-n_7}{d_{25}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_3, \ell, k_4\}] == -\frac{n_5}{d_5} - \frac{n_3-n_4+n_5}{d_2} - \frac{n_5-n_6}{d_{26}} - \frac{n_5-n_6}{d_{35}} - \frac{n_5-n_6}{d_{56}} - \frac{n_3+n_5-n_6}{d_1} - \frac{n_3+n_5-n_6-n_7}{d_{25}} - \frac{n_3+n_5-n_6-n_7}{d_{28}} - \frac{n_3+n_5-n_6-n_7}{d_{55}} - \frac{n_5-n_9+n_{10}}{d_{19}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_4, k_3, \ell\}] == \frac{n_3-n_4}{d_{29}} + \frac{n_3-n_4+n_5}{d_2} - \frac{n_4-n_6}{d_{27}} + \frac{n_3+n_5-n_6-n_7}{d_{28}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, k_4, \ell, k_3\}] == -\frac{n_3}{d_3} - \frac{n_3-n_4}{d_{29}} - \frac{n_3-n_4}{d_{31}} - \frac{n_3-n_4}{d_{48}} - \frac{n_3-n_4+n_5}{d_2} - \frac{n_3+n_5-n_6}{d_1} - \frac{n_3+n_5-n_6-n_7}{d_{25}} - \frac{n_3+n_5-n_6-n_7}{d_{28}} - \frac{n_3+n_5-n_6-n_7}{d_{47}} - \frac{n_3-n_7+n_9}{d_{13}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, \ell, k_3, k_4\}] == \frac{n_5}{d_5} + \frac{n_3-n_4+n_5}{d_2} - \frac{n_4-n_6}{d_{27}} - \frac{n_4-n_6}{d_{33}} - \frac{n_4-n_6}{d_{52}} + \frac{n_6}{d_6} + \frac{n_3+n_5-n_6-n_7}{d_{28}} + \frac{n_3+n_5-n_6-n_7}{d_{55}} + \frac{n_3+n_5-n_6-n_7}{d_{60}} + \frac{n_6-n_7+n_{10}}{d_{23}} + \frac{n_5-n_9+n_{10}}{d_{19}} + \frac{n_6-n_9+n_{10}}{d_{20}}$$

$$\mathbb{I}[\{-\ell, k_1, k_2, \ell, k_4, k_3\}] == \frac{n_3}{d_3} + \frac{n_4}{d_4} + \frac{n_4-n_6}{d_{27}} + \frac{n_4-n_6}{d_{33}} + \frac{n_4-n_6}{d_{52}} + \frac{n_3+n_5-n_6}{d_1} + \frac{n_3+n_5-n_6-n_7}{d_{25}} + \frac{n_3+n_5-n_6-n_7}{d_{47}} + \frac{n_3+n_5-n_6-n_7}{d_{53}} + \frac{n_3-n_7+n_9}{d_{13}} + \frac{n_4-n_7+n_9}{d_{14}} + \frac{n_4-n_7+n_{10}}{d_{17}}$$

$$\mathbb{I}[\{-\ell, k_1, k_3, k_2, k_4, \ell\}] == \frac{n_3}{d_3} + \frac{n_3-n_4}{d_{31}} - \frac{n_5-n_6}{d_{26}} + \frac{n_7-n_9}{d_{30}}$$

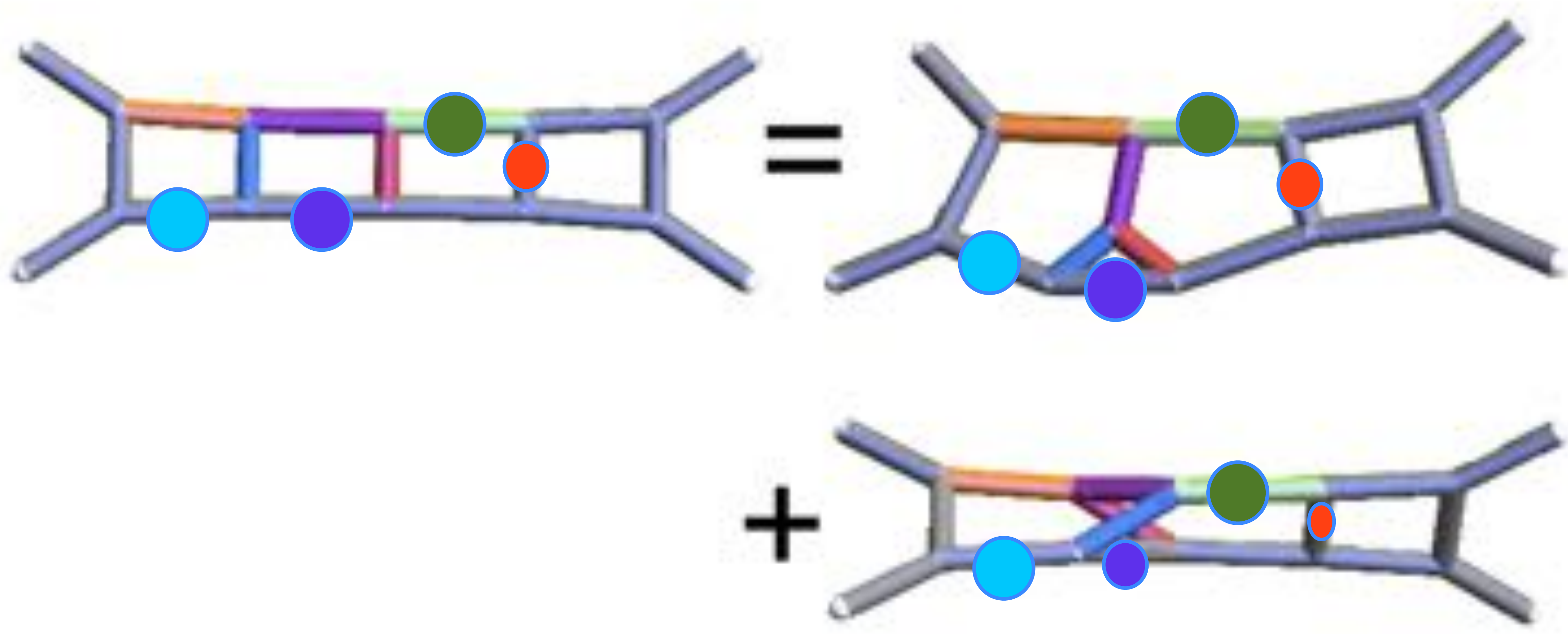
CAN INVERT OFF-SHELL: $\ell^2 \neq 0$

7 independent CO Pre-Intg

Different than the 1-particle cut: $\ell^2 = 0$

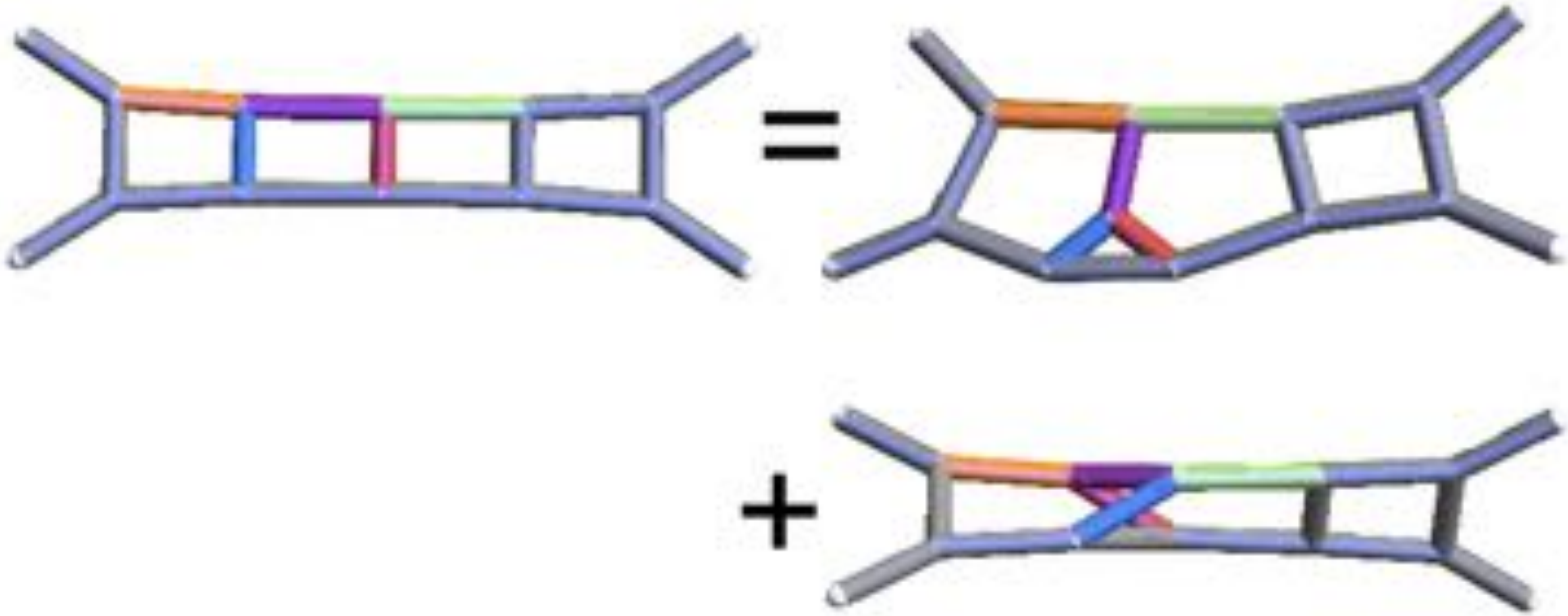
6 independent CO 1-particle cuts from (m-3)! tree-level!

After Jacobi, now have a color-kinematic satisfying representation at loop level -- no ansatz.



Asymmetric graphs can have Jacobi's imposed linearly on all edges but L

Conjecture: this is sufficient for double-copy to hold



Verification: Gravity amplitude must be checked on a spanning set of cuts by symmetrizing into symmetric functional representation.

Explicitly verified: 1 loop 4-pt for N<=4 SYM
2 loop 4-pt for N=4 SYM

Exploring constructive all-multiplicity 1-loop proofs now

Summary: Presented path forward to find C/K satisfying representations without an ansatz.

There is a cautionary note, this way forward involves increasing the redundancy of graph descriptions — no free lunch, but at least a bounded complexity problem.

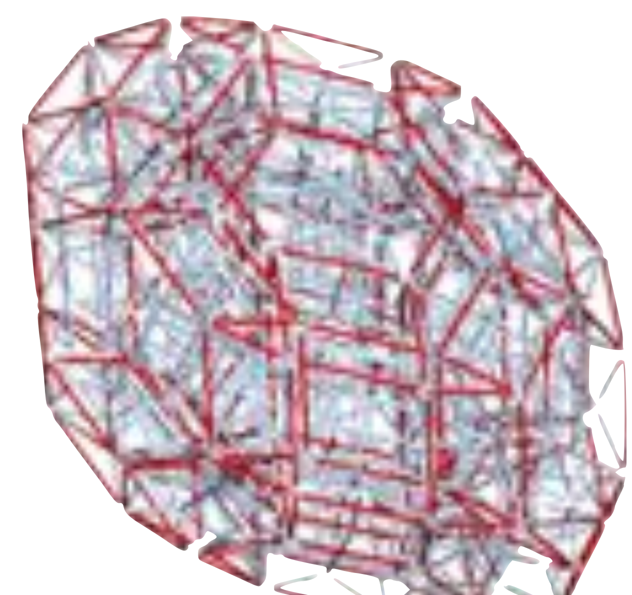
The HOPE

Should be a spring board to a description that starts collapsing the redundancy—we know it is possible in many situations!

May be an avenue to recycle formal all-multiplicity tree-level insight into all multiplicity loop-level insight

Will be a vehicle to get more c/k data at lower-loops/higher multiplicity in theories with less SUSY

Happy to help you play these games with your own non-planar integrands!



6. Classical Solutions

This is a scattering celebration, but I do want to take a second to mention the potential importance of a deeper understanding of classical solutions.

Given all tree-level doubly-copy relations between YM and Gravity, can we expect classical solutions to GR+matter EOM as a double copy of solutions to YM+matter EOM?

(See also work of Saotome & Akhoury and combinations of Anastasiou, Borsten, Duff, Hughes, Nagy)

Monteiro, O'Connell, and White, along with increasing list of collaborators are amassing evidence that the answer is **yes**, at least for a certain class of solutions.

Monteiro, O'Connell, White '14

Luna, Monteiro, O'Connell, White '15

Luna, Monteiro, Nicholson, O'Connell, White (to appear)

Monteiro, O'Connell, and White

3-pt Scattering Amplitude

$$\frac{\mathbf{c(g)n(g)}}{\mathbf{d(g)}} \longrightarrow \frac{\mathbf{n(g)n(g)}}{\mathbf{d(g)}}$$

Double Copy

Classical Solutions (in a special class called Kerr-Schild)

$$\mathbf{A^a_m u} = \mathbf{c^a k}_\nu \phi \longrightarrow \mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \mathbf{k}_\mu \mathbf{k}_\nu \phi$$

Double Copy

Schwarzschild

$$\mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_\mu\mathbf{k}_\nu$$

$$\mathbf{k}_\mu = \{\mathbf{1}, \hat{\mathbf{r}}\}$$

Monteiro, O'Connell, and White

Schwarzschild

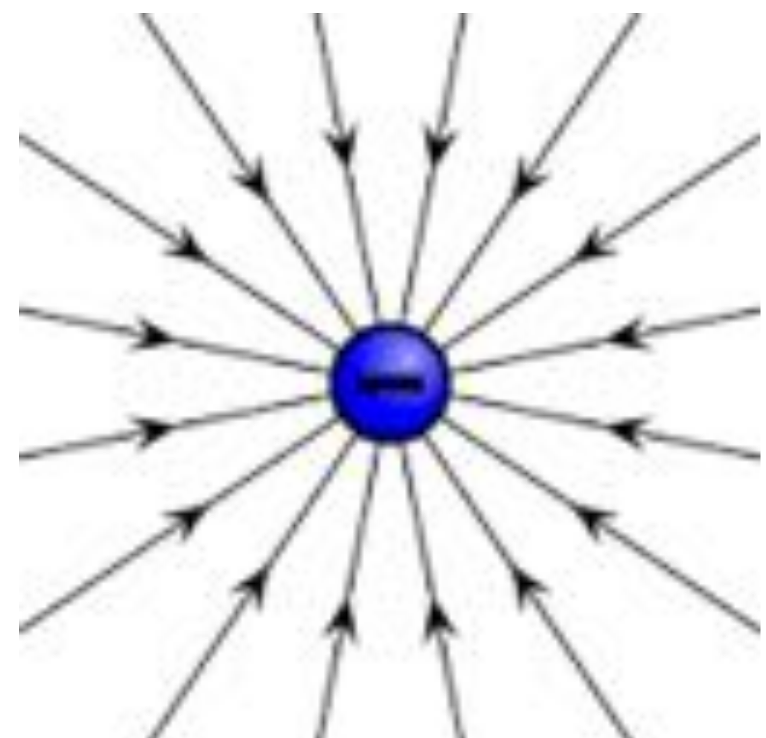
$$\mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_\mu \mathbf{k}_\nu$$

$$\mathbf{k}_\mu = \{\mathbf{1}, \hat{\mathbf{r}}\}$$

The double copy of

$$\mathbf{A}_\mu = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_\mu$$

abelianized point charge



Monteiro, O'Connell, and White

Monteiro, O'Connell, and White

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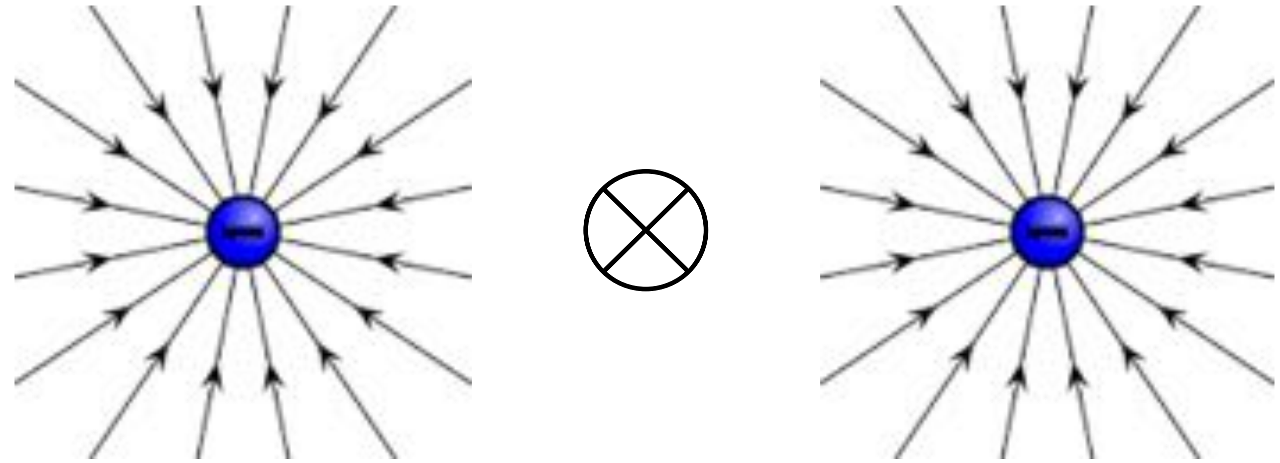
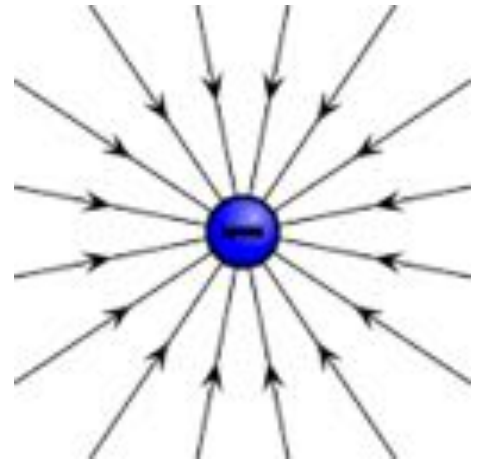


Natural question:

What process double copies to Hawking radiation?

Monteiro, O'Connell, and White

$=$



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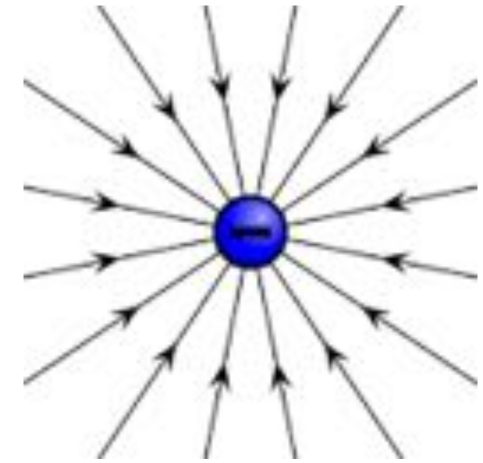

Natural question:

What process double copies to Hawking radiation?

Suggestive answer:

Schwinger pair production

Torroba, & JJMC (to appear)

Classical gravity is a Double Copy?

Remind you of some of the double-copy positives:

+ Constrained solutions => can exploit for technical simplicity in prediction

+ Web of relationships between theories

Open question: how far can this go?