Superstring Amplitudes: MHV and Beyond

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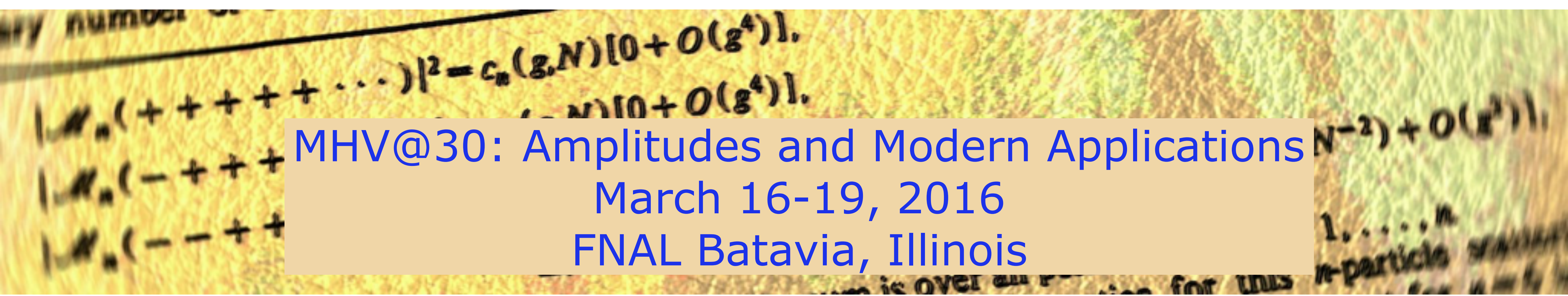

MHV@30: Amplitudes and Modern Applications
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Parke-Taylor amplitude (1986):

$$A_{YM}(1^-,2^-,3^+,\ldots,N^+) = i\ \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\cdot\ldots\cdot\langle N1\rangle}$$

Corrections from string theory ?:

$\alpha' \sim M_{\rm string}^{-2}$

$$A(1^-,2^-,3^+,\ldots,N^+) = \left(1-\frac{1}{2}\zeta_2\ Q_N\right)\ A_{YM}(1^-,2^-,3^+,\ldots,N^+) + \mathcal{O}({\alpha'}^3)$$

$$Q_4 = s_1 s_2$$

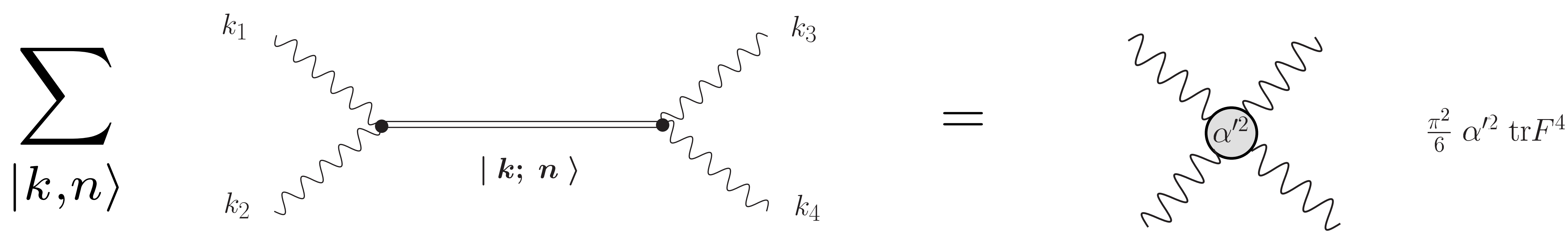
$$Q_5 = s_1 s_2 + s_2 s_3 + s_3 s_4 + s_4 s_5 + s_5 s_1 + \epsilon(1,2,3,4)$$

$$Q_6 = \{s_1 s_2\} - \{s_1 s_4\} + \{t_1 t_2\} + \sum_{k<l<m<n<6} \epsilon(k,l,m,n)$$

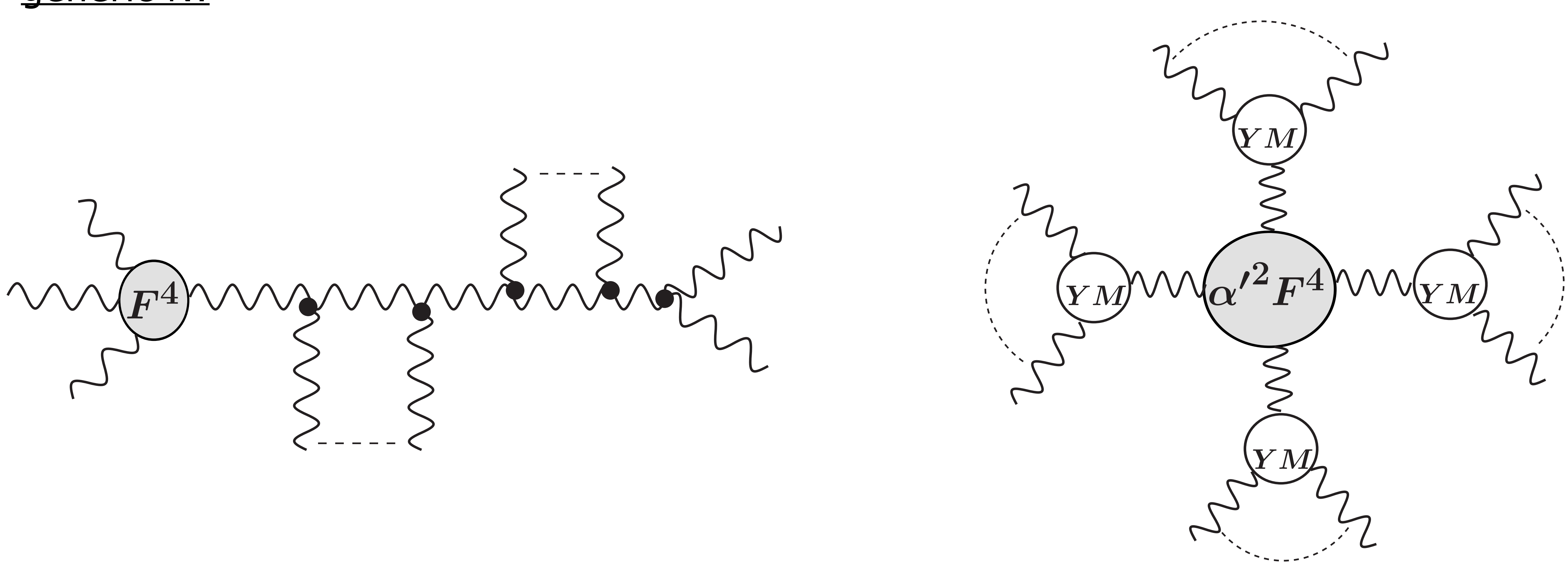
$$\vdots$$

St.St., Taylor (2006)

N=4:


$\sum_{|k,n\rangle}$
k_1
k_2
k_3
k_4
$|\, \boldsymbol{k};\ \boldsymbol{n}\,\rangle$
$=$
α'^2
$\frac{\pi^2}{6}\ \alpha'^2\ \mathrm{tr}F^4$

generic N:


F^4
YM
YM
$\alpha'^2 F^4$
YM
YM

N-particle scattering kinematics: $k_i^2 = 0, \quad k_1 + \ldots k_N = 0$

$$[\![i]\!]_n = \alpha' \; (k_i + k_{i+1} + \cdots + k_{i+n})^2 \;, \qquad n = 1, \ldots, \lfloor \frac{N}{2} \rfloor - 1, \quad k_{i+N} \equiv k_i$$

$$\epsilon(i,j,m,n) = 4i\,\alpha'^{\,2}\,\epsilon_{\alpha\beta\mu\nu}\,k_i^\alpha k_j^\beta k_m^\mu k_n^\nu = [ij]\langle jm\rangle[mn]\langle ni\rangle - \langle ij\rangle[jm]\langle mn\rangle[ni]$$

$$Q_N = \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor - 1} \{\, [\![1]\!]_k [\![2]\!]_k \} - \sum_{k=3}^{\lfloor \frac{N}{2} \rfloor - 1} \{\, [\![1]\!]_k [\![2]\!]_{k-2} \} + \sum_{k<l<m<n<N} \epsilon(k,l,m,n)$$

$$- \begin{cases} \{\, [\![1]\!]_{\frac{N}{2}-2} [\![\frac{N}{2}+1]\!]_{\frac{N}{2}-2} \} & N > 4, \text{ even} \\ \{\, [\![1]\!]_{\frac{N-5}{2}} [\![\frac{N+1}{2}]\!]_{\frac{N-3}{2}} \} & N > 5, \text{ odd} \end{cases}$$

uniquely fixed by:

- cyclic invariance
- soft-limit
- collinear limit

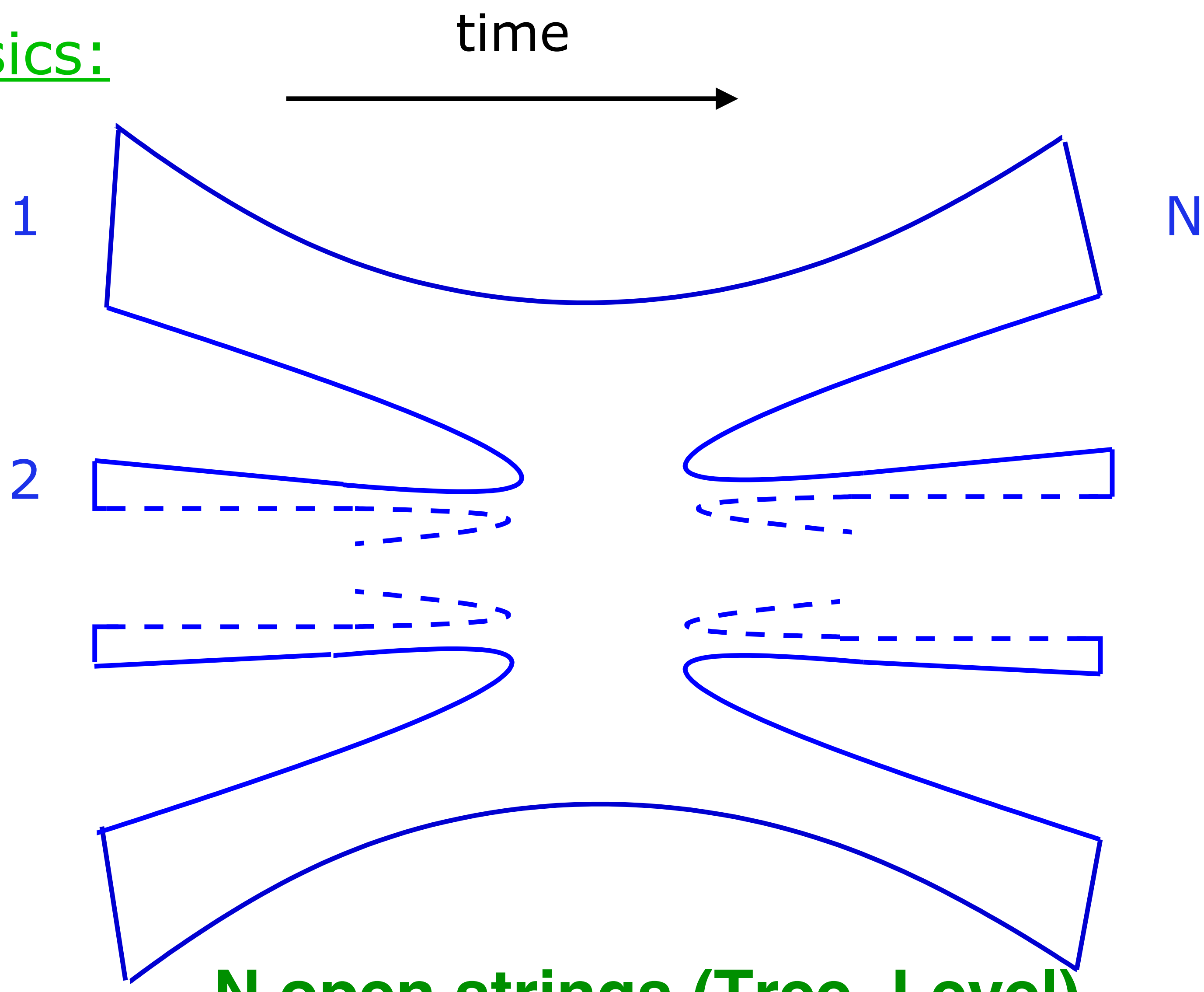
this form agrees with **planar QCD all-plus amplitude** $A_{N;1}$

(no cuts and multi-particle poles appearing)

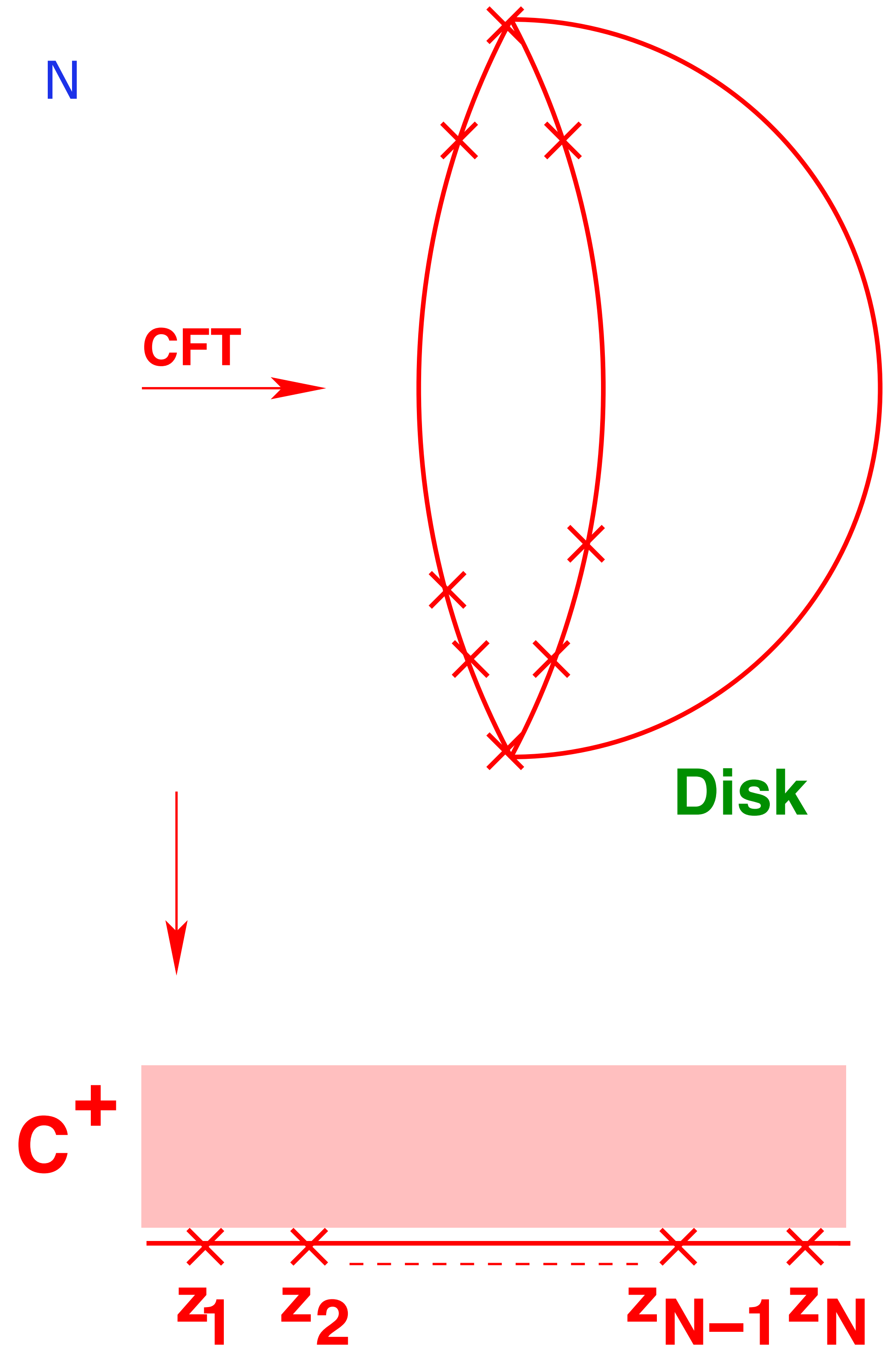
one-loop all-plus FT = tree-level MHV STTH

St.St., Taylor (2006)

Physics:



N open strings (Tree–Level)



$$\mathcal{A}(1,2,\ldots,N;\alpha')$$

from monodromies of world-sheet:

(N-3)! independent open string amplitudes

St.St., arXiv:0907.2211

Bjerrum-Bohr, Damgaard, Vanhove, arXiv:0907.1425

Full-fledged MHV N-gluon amplitude in superstring theory

$$A(1,\ldots,N) = \frac{1}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\left(\prod_{k=4}^{N}\langle 2k\rangle\right)^{-2}\int d\mu_N(z,s)\sum_{\text{trees}}\ \prod_{\text{edges}}\frac{s_{ij}}{z_{ij}\ z'_{ij}}$$

St.St., Taylor (2012)

$$z'_{ij} = z'_i - z'_j = \frac{\langle ij\rangle}{\langle 2i\rangle\langle 2j\rangle}$$

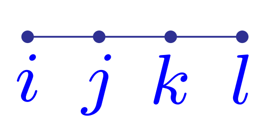
$$z_{ij} = z_i - z_j = \text{difference vertex operator positions on world-sheet}$$

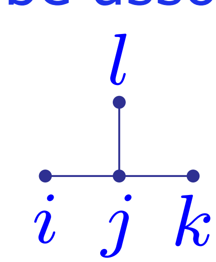
Koba-Nielsen factor

$$\int d\mu_N(z,s) := \int_1^\infty dz_4 \ldots \int_{z_{N-1}}^\infty dz_N \prod_{2\le k<l\le N} |z_{kl}|^{s_{kl}}, \quad s_{kl} = 2\alpha'\ p_k p_l$$

full graphical description in terms of world-sheet trees

to each product of rational functions a tree graph can be associated:

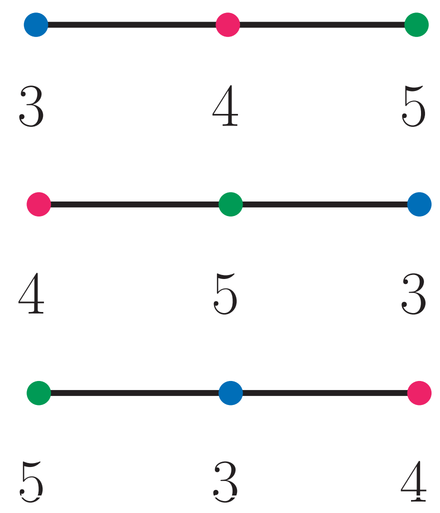
$$\frac{1}{z_{ij}z_{jk}z_{kl}} =$$ 

$$\frac{1}{z_{ij}z_{jk}z_{jl}} =$$ 

sum is over all

inequivalent connected tree-graphs with vertices labelled by 3,…,N

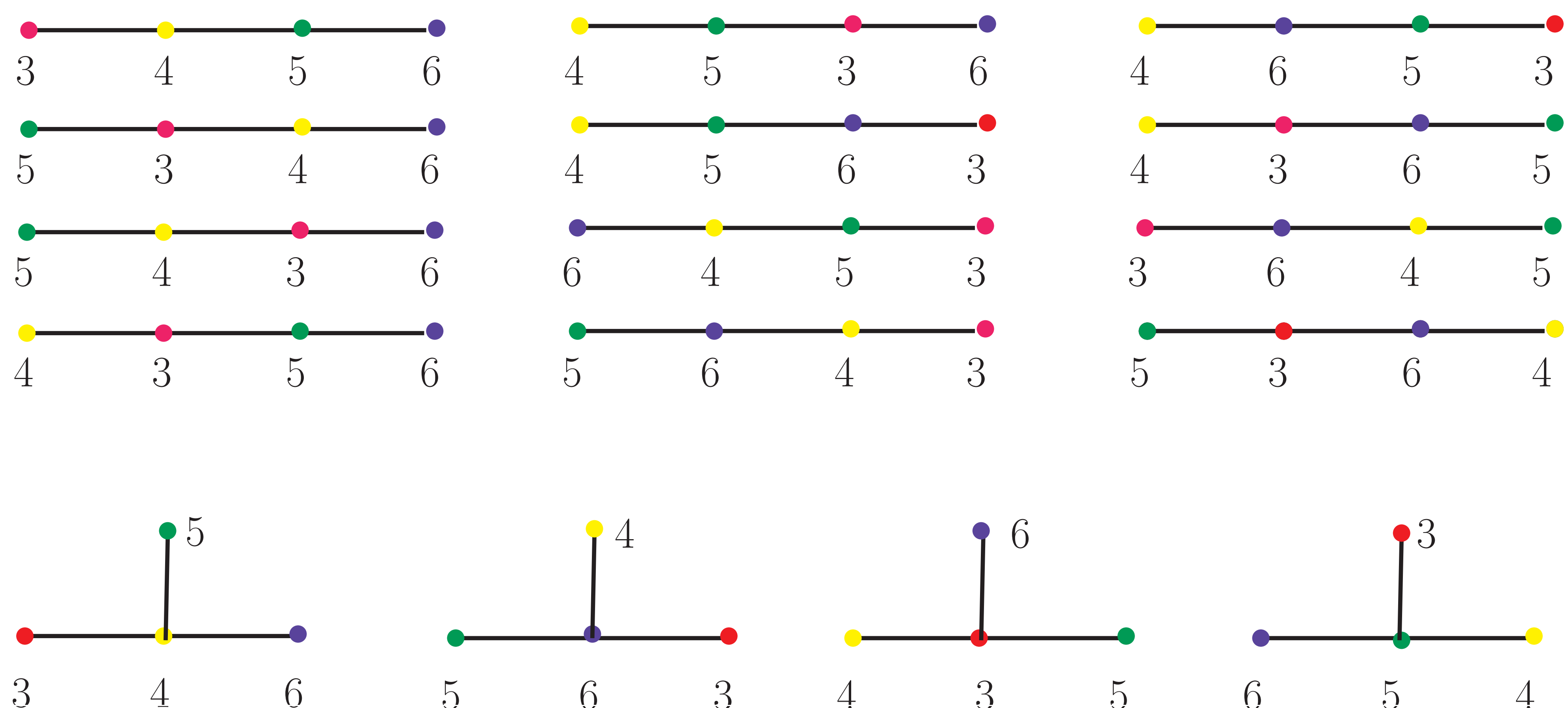
N=5:



Cayley graphs

$(N-2)^{N-4}$

N=6:



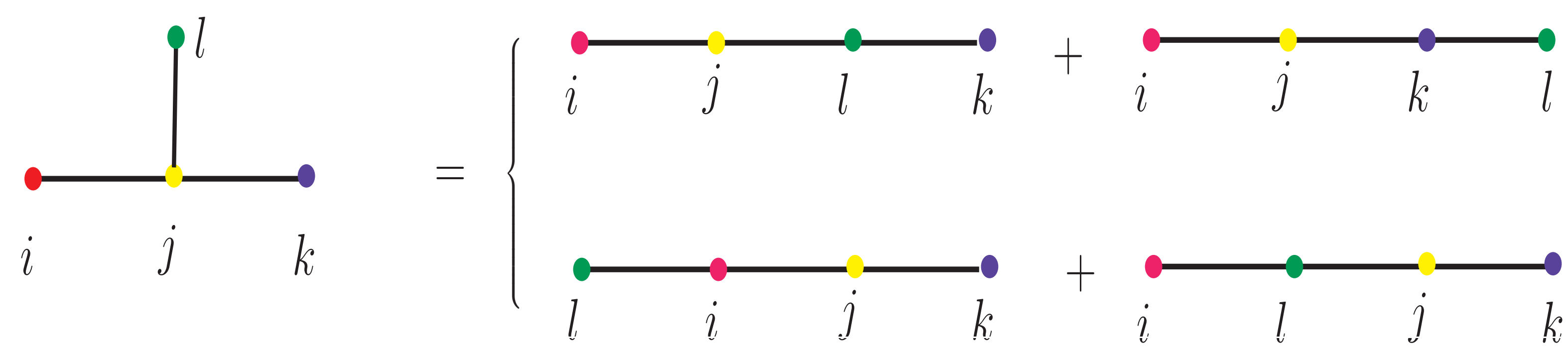
Partial fractioning:

$$\frac{1}{z_{ij}z_{jk}}+\frac{1}{z_{ik}z_{kj}}+\frac{1}{z_{ij}z_{ki}} = 0 = \frac{1}{z'_{ij}z'_{jk}}+\frac{1}{z'_{ik}z'_{kj}}+\frac{1}{z'_{ij}z'_{ki}}$$

Partial fraction decomposition on rational functions

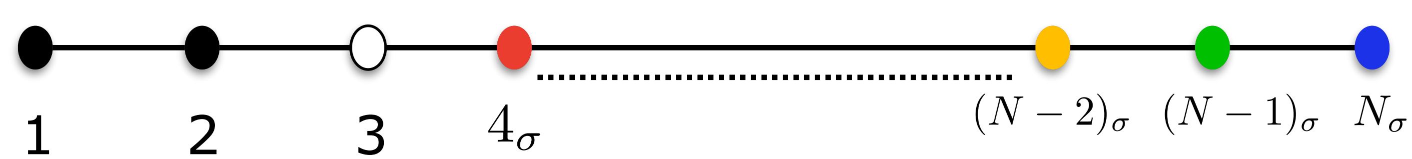
→ relations between trees:

$$\frac{1}{z_{ij}z_{jk}z_{jl}} = \begin{cases} \frac{1}{z_{ij}}\left(\frac{1}{z_{jl}z_{lk}}+\frac{1}{z_{jk}z_{kl}}\right) \\ -\left(\frac{1}{z_{li}z_{ij}}+\frac{1}{z_{il}z_{lj}}\right)\frac{1}{z_{jk}} \end{cases}$$



→ Hamilton graphs

$(N-3)!$



$$\left(z_{34_\sigma}z_{5_\sigma 6_\sigma}\cdot\ldots\cdot z_{(N-1)_\sigma N_\sigma}\right)^{-1}$$

Full-fledged MHV N-gluon amplitude in superstring theory

$$A(1,\ldots,N) = \frac{1}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \int d\mu_N(z,s) \sum_{\mathcal{P}} \prod_{k=4}^{N} \frac{\langle 2|3+\ldots+(k-1)|k]}{\langle 2k\rangle} \frac{1}{z_{(k-1)k}}$$

with $(N-3)!$ permutations $\mathcal{P}$ of $4,5,\ldots,N$

St.St., Taylor (2012)

$$\langle 2|3+\ldots+(k-1)|k] = \langle 23\rangle[3k]+\ldots+\langle 2(k-1)\rangle[(k-1)k]$$

N=4:

$$A(1,2,3,4) = \frac{1}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \int_1^\infty dz_4 \; |z_{24}|^{s_{24}} \; |z_{34}|^{s_{34}} \; \frac{1}{z_{34}} \; \frac{\langle 2|3|4]}{\langle 24\rangle}$$

$$= \frac{1}{\langle 12\rangle\langle 31\rangle\langle 34\rangle\langle 24\rangle} \; s \; \frac{\Gamma(s)\;\Gamma(u)}{\Gamma(s+u)} \xrightarrow{\alpha'\to 0} \frac{1}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$s = s_{12} = s_{34}$$

$$u = s_{14} = s_{23}$$

$$z_1 = -\infty,\; z_2 = 0,\; z_3 = 1$$

Sugra vs. open superstring theory

Open superstring scattering:

$$A(1,\ldots,N) = \frac{1}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} \int d\mu_N(z,s) \sum_{\mathcal{P}} \prod_{k=4}^{N} \frac{\langle 2|3+\ldots+(k-1)|k]}{\langle 2k\rangle} \frac{1}{z_{(k-1)k}}$$

$$z_{ij} = z_i - z_j = \text{difference vertex operator positions on world-sheet}$$

Koba-Nielsen factor $\int d\mu_N(z,s)$

Graviton field-theory scattering:

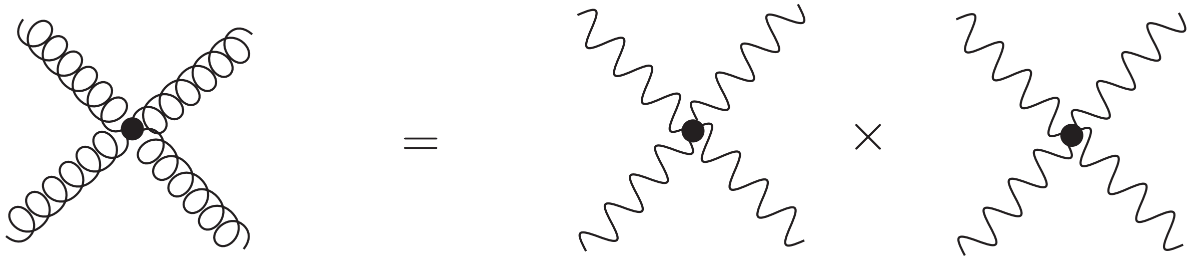
$$\mathcal{M}_{FT}(1,\ldots,N) = \frac{1}{\langle 12\rangle^2\langle 23\rangle^2\langle 31\rangle^2} \left(\prod_{k=4}^{N} \langle 1k\rangle\right)^{-2} \sum_{\mathcal{P}} \prod_{k=4}^{N} \frac{\langle 2|3+\ldots+(k-1)|k]}{\langle 2k\rangle} \frac{1}{z_{(k-1)k}}$$

$$z_{ij} = \frac{\langle ij\rangle}{\langle 1i\rangle\langle 1j\rangle}$$

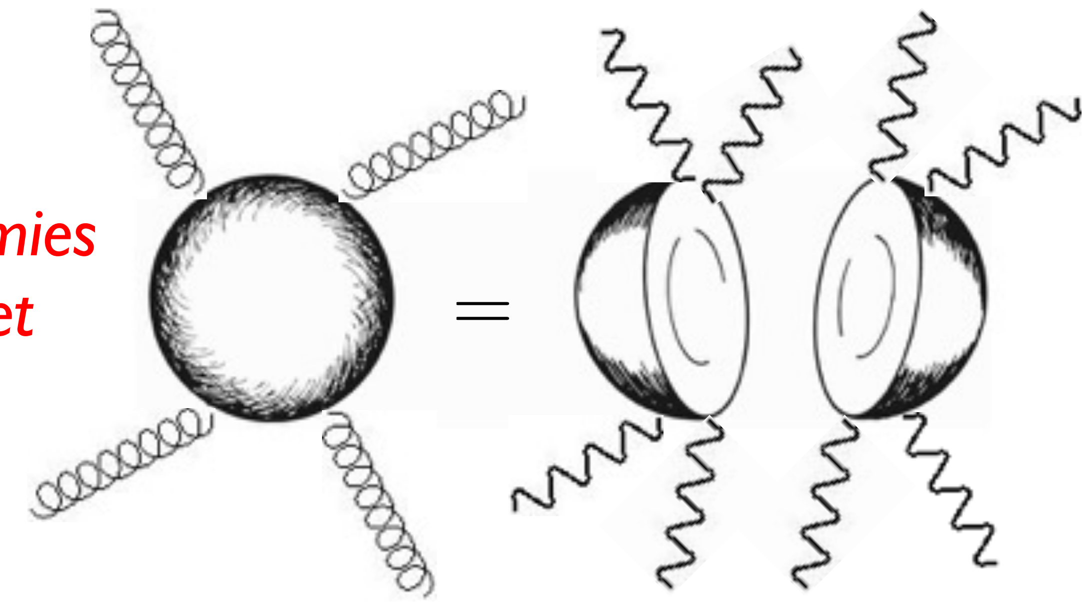
Mason, Skinner, arXiv:0808.3907 [hep-th]

striking interplay between positions and spinors

Kawai-Lewellen-Tye (KLT) relations



from monodromies of world-sheet



$$
\begin{aligned}
\mathcal{M}_{FT}(1,\ldots,4) \quad &= \quad s_{12}\; A_{YM}(1,2,3,4)\; \tilde{A}_{YM}(1,2,4,3) \\
\text{graviton amplitude} \quad &= \quad (\text{gauge amplitude}) \;\times\; (\text{gauge amplitude})
\end{aligned}
$$

Supergravity graviton N-point tree-level amplitude:

$$\mathcal{M}_{FT}(1,\ldots,N) = (-1)^{N-3}\,\kappa^{N-2} \sum_{\sigma\in S_{N-3}} A_{YM}(1,\sigma(2,3,\ldots,N-2),N-1,N)$$

$$\times \sum_{\rho\in S_{N-3}} S[\rho|\sigma]\;\tilde{A}_{YM}(1,\rho(2,3,\ldots,N-2),N,N-1)$$

S = KLT kernel

$$S[\rho|\sigma] \;:=\; S[\,\rho(2,\ldots,N-2)\,|\,\sigma(2,\ldots,N-2)\,] \;=\; \prod_{j=2}^{N-2}\Big(s_{1,j_\rho} + \sum_{k=2}^{j-1}\theta(j_\rho,k_\rho)\,s_{j_\rho,k_\rho}\Big)$$

$$s_{ij} = \alpha'(k_i+k_j)^2$$

Bern, Dixon, Perelstein, Rozowsky (1998)

Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove (2010)

Type I gauge amplitude:

Mafra, Schlotterer, St.St. (2011)
Broedel, Schlotterer, St.St. (2013)

$$A(\pi) = (-1)^{N-3} \sum_{\sigma \in S_{N-3}} \sum_{\rho \in S_{N-3}} Z_\pi(\rho)\ S[\rho|\sigma]\ A_{YM}(\sigma)$$

fundamental
world-sheet
disk integrals

$$Z_\pi(\rho) \ := \ Z_\pi(1, \rho(2, \ldots, N-2), N, N-1)$$

$$= \ V_{\text{CKG}}^{-1} \left(\prod_{j=1}^{N} \int_{D(\pi)} dz_j \right) \frac{\prod_{i<j}^{N} |z_{ij}|^{s_{ij}}}{z_{1\rho(2)} z_{\rho(2),\rho(3)} \ldots z_{\rho(N-3),\rho(N-2)} z_{\rho(N-2),N} z_{N,N-1}, z_{N-1,1}}$$

iterated real integral on $\mathbf{RP}^1 \backslash \{0, 1, \infty\}$

$$D(\pi) \ = \ \{ z_j \in \mathbf{R} \mid 0 < z_{\pi(2)} < \ldots < z_{\pi(N-2)} < 1 \}$$

$$s_{ij} = \alpha'(k_i + k_j)^2 = 2\alpha'\ k_i k_j$$

disk integrals behave like SYM subamplitudes: fulfill KK & BCJ relations etc.

compare with gravitational field-theory amplitude:

Taylor, St.St. (2013)

$$\tilde{A}_{YM}(\rho) \simeq Z_\pi(\rho)$$

$$\mathcal{M}_{FT} = (-1)^{N-3} \sum_{\sigma \in S_{N-3}} \sum_{\rho \in S_{N-3}} \tilde{A}_{YM}(\rho)\ S[\rho|\sigma]\ A_{YM}(\sigma)$$

gives rise to superstring/supergravity Mellin correspondence

EYM Amplitudes from String Theory

basic building blocks for BCFW recursion relations:

$$A(1^-, 2^-, 3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$$

$$A(1^{--}, 2^{--}, 3^{++}) = \frac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 31\rangle^2} \qquad A(1^+, 2^-, 3^{--}) = \frac{\langle 23\rangle^4}{\langle 12\rangle^2}$$

standard Feynman diagram computation or BCFW recursion relations yields, e.g.:

$$A(1^+, 2^+, 3^-; q^{--}) = \frac{\langle 3q\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

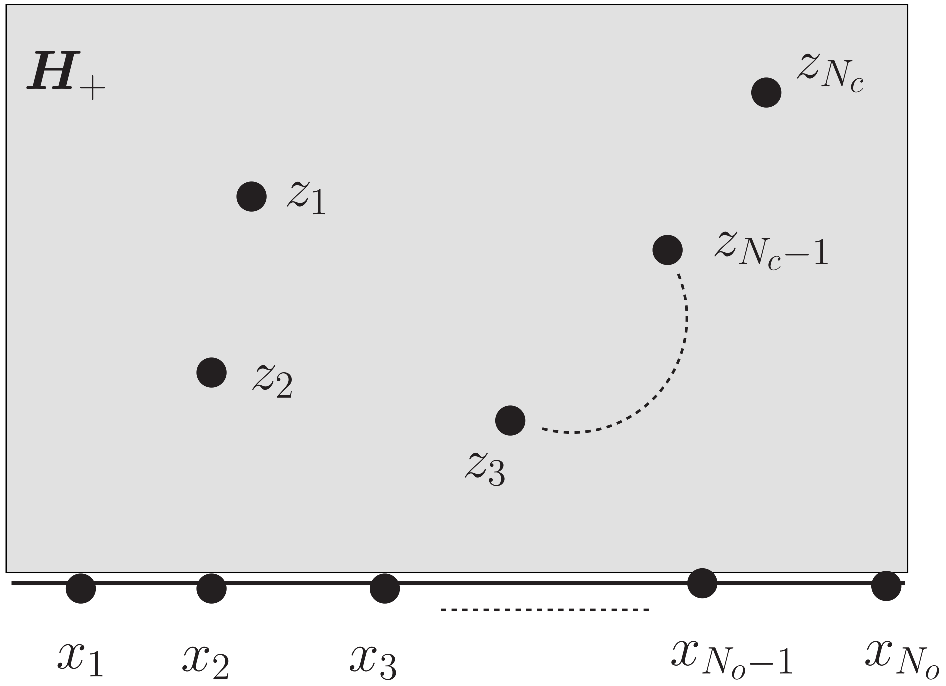
Mixed amplitudes involving open and closed strings:

<u>*"Doubling trick":*</u>

- *convert disk correlators to the **standard holomorphic** ones by extending the fields to the **entire complex plane**.*

<u>*"KLT trick":*</u>

- *integration over complex positions of closed string states can be **disentangled** into real ones by introducing **monodromy phases***



→ *monodromy problem on the complex plane*

N_o open & N_c closed strings: N_o+2N_c- point **pure** open string amplitude

St.St. arXiv:0907.2211

$N_c = 1$

$$A(1,2,\ \dots,N{-}2;q)$$

$$= \sum_{l=2}^{\lceil \frac{N}{2}\rceil -1} \sum_{i=2}^{l} \sin\left(\pi \sum_{j=i}^{l} s_{j,N-1}\right) A(1,\dots,i-1,N,i,\dots,l,N-1,l+1,\dots,N-2)$$

$$+ \sum_{l=\lceil \frac{N}{2}\rceil}^{N-3} \sum_{i=l+1}^{N-2} \sin\left(\pi \sum_{j=l+1}^{i} s_{j,N-1}\right) A(1,\dots,l,N-1,l+1,\dots,i,N,i+1,\dots,N-2)$$

$(\lceil \frac{N}{2} \rceil - 2)\ (\lfloor \frac{N}{2} \rfloor - 1)$ terms

$s_{i,j} \equiv s_{ij} = 2\alpha' k_i k_j$

St.St., Taylor (2014)

relations between
amplitudes involving **open & closed strings** and
pure open string amplitudes

<u>Examples:</u>

$$A(1,2,3;q) = \sin(\pi s_{24})\ A(1,5,2,4,3)\ ,$$

$$A(1,2,3,4;q) = \sin(\pi s_{25})\ A(1,6,2,5,3,4) + \sin(\pi s_{45})\ A(1,2,3,5,4,6)\ ,$$

$$A(1,2,3,4,5;q) = \sin(\pi s_{26})\ A(1,7,2,6,3,4,5) + \sin(\pi s_{36})\ A(1,2,7,3,6,4,5)$$
$$+ \sin[\pi(s_{36}+s_{26})]\ A(1,7,2,3,6,4,5) + \sin(\pi s_{56})\ A(1,2,3,4,6,5,7)$$

(in collinear limit: $k_{N-1} = \frac{1}{2}q,\ k_N = \frac{1}{2}q$ *)*

<u>take field-theory limit:</u>

yields **Einstein-Yang-Mills**
for any kinematical configuration

"graviton appears as a pair of collinear gauge bosons"

St.St., Taylor (2014)

$$A_{EYM}(1^+,2^+,3^-;q^{--}) = \pi\ s_{24}\ A_{\rm YM}(1^+,5^-,2^+,4^-,3^-)$$

with SYM amplitude:

$$A_{\rm YM}(1^+,5^-,2^+,4^-,3^-) = 4\ \frac{[12]^4}{[1q][q3][13][2q]^2}$$

MHV case: Bern, De Freitas, Wong, arXiv:hep-th/9912033
from *squares* of open string amplitudes (*heterotic string*)

generalization to arbitrary collinear configuration

$$A(1,2,3;q_1,q_2) = \frac{\kappa\,(1-x)}{g^2}\; s_{24}\; A(1,5,2,4,3)\;,$$

$$A(1,2,3,4;q_1,q_2) = \frac{\kappa\,(1-x)}{g^2}\left\{\, s_{25}\; A(1,6,2,5,3,4) + s_{45}\; A(1,2,3,5,4,6)\,\right\},$$

$$A(1,2,3,4,5;q_1,q_2) = \frac{\kappa\,(1-x)}{g^2}\left\{\, s_{26}\; A(1,7,2,6,3,4,5) + s_{36}\; A(1,2,7,3,6,4,5)\right.$$

$$\left. + (s_{36}+s_{26})\; A(1,7,2,3,6,4,5) + s_{56}\; A(1,2,3,4,6,5,7)\,\right\}$$

graviton is replaced by two gluons
in **arbitrary** collinear configurations

$$q_1 = k_{N-1} = x\,q,$$
$$q_2 = k_N = (1-x)\,q$$

St.St., Taylor (2015)

gives rise to some underlying
gauge structure
in quantum gravity

more interesting results to appear soon

with T.R. Taylor

Concluding remarks

- structure of graviton scattering reminiscent from open superstring scattering
- graviton scattering **unified** into gauge amplitudes
- growing set of **interconnections** between open & closed amplitudes with gauge theory and supergravity amplitudes
- possible **dual description** of perturbative open superstring theory ?

Graviton amplitudes from gauge amplitudes

express N-graviton amplitude in **Einstein's gravity**
as collinear limits of
certain linear combinations of **pure SYM amplitudes**
in which each graviton is represented by two gauge bosons

no string theory !
but motivated from string theory

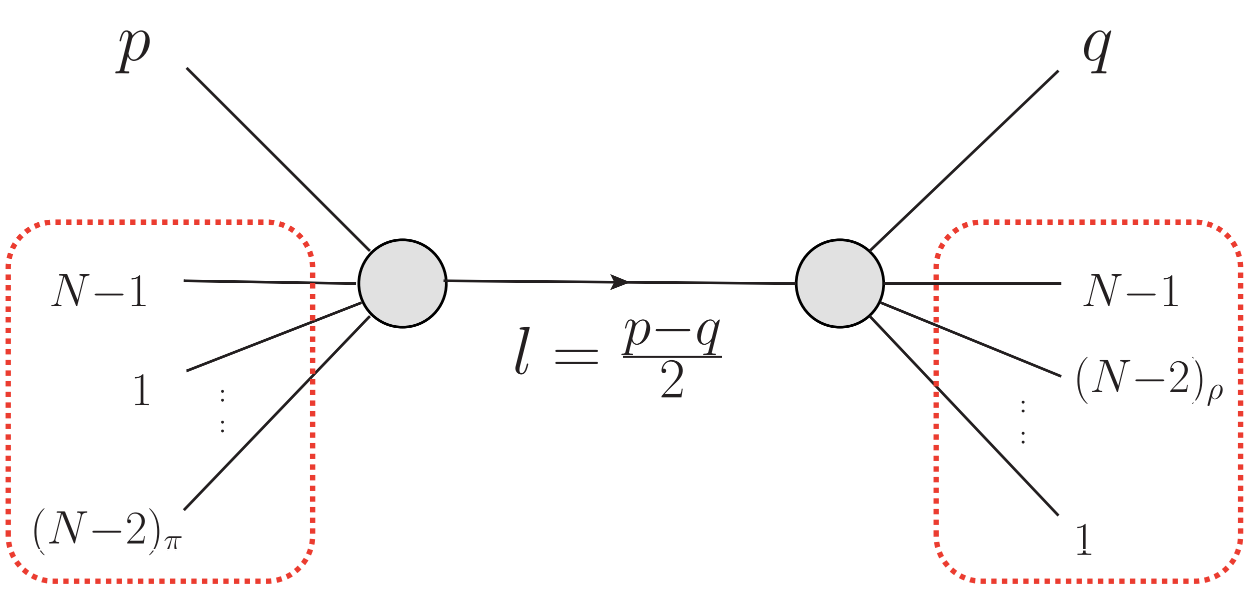
$$A_E[k_1,\lambda_1;\ldots;k_{N-1},\lambda_{N-1};k_N=p+q,\lambda_N=+2]=\lim_{[pq]\to 0}\left(\frac{1}{2x}\right)^4\frac{[pq]}{\langle pq\rangle}s_{pq}^2$$

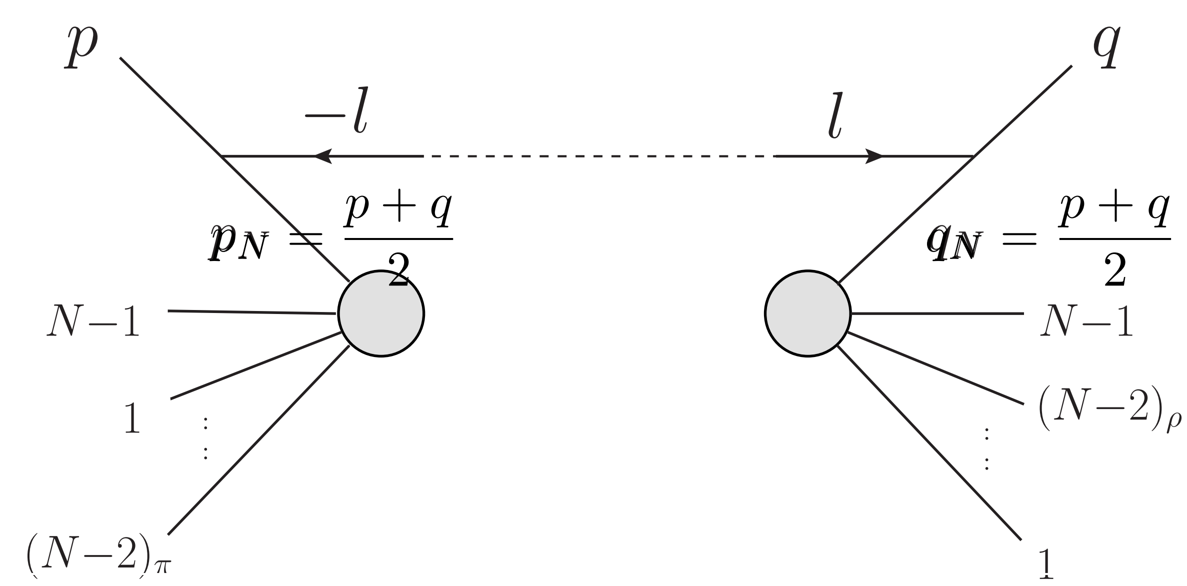
$$\times\sum_{\pi,\rho\in S_{N-3}}S[\pi|\rho]\;A_{YM}[p,N-1,1,\pi(2,3,\ldots,N-2),1,\rho(2,\ldots,N-2),N-1,q]$$

(2N-2 gluons become collinear without producing poles)

St.St., Taylor (2014)

Proof: contributions from factorization on **triple** pole $s_{pq}^3 \sim (p-q)^6$





$$A_{YM}[p, N-1, 1, \pi(2,3,\ldots,N-2), 1, \rho(2,\ldots,N-2), N-1, q] \ \rightarrow \ \left(\frac{4}{s_{pq}}\right)^3 \times$$

$$\times\ A_{YM}[p^+, -l^-, -p_N^-] \times A_{YM}[p_N, \mu_N = +1; N-1, 1, \pi(2,3,\ldots,N-2)]$$

$$\times\ A_{YM}[1, \rho(2,\ldots,N-2), N-1; q_N, \nu_N = +1] \times A_{YM}[q^-, l^+, -q_N^-]$$

$$A_{YM}[p^+, -l^-, -p_N^-] = \frac{x^3}{2}\langle pq\rangle,$$

$$A_{YM}[q^-, l^+, -q_N^-] = \frac{x}{2}\langle pq\rangle$$

yields:

$$A_E[k_1, \lambda_1; \ldots; k_{N-1}, \lambda_{N-1}; k_N, \lambda_N = +2]$$

$$= \sum_{\pi,\rho \in S_{N-3}} S[\pi|\rho]\ A_{YM}[p_N, \mu_N = +1; N-1, 1, \pi(2,3,\ldots,N-2)]$$

$$\times A_{YM}[1, \rho(2,\ldots,N-2), N-1; q_N, \nu_N = +1]$$